Physical Sciences and Technology: Second-round Sample Tasks for the Open Doors Postgraduate Track

You will be asked to complete 35 tasks, including:

- 21 entry-level tasks, each correct answer worth 1 point;
- 11 intermediate-level tasks, each correctly answered task worth 3 or 4 points;
- 3 advanced tasks (constructed response), each correctly completed task valued at 15 points.

Evaluation criteria and standard answers are provided for the advanced tasks requiring constructed responses.

Mechanics

Task 1
Entry level (1 point)

An object with a mass of 100 kg is sliding down an inclined plane with a constant velocity at an angle of 30°. What is the magnitude of the total force acting on the object? The answers are given in kilo-newtons.

a) 0
b) 1
c) 2
d) 4

Answer: a.

Task 2
Entry level (1 point)

The magnitude of the total force, acting on a particle, depends on the displacement s of the particle, given by F(s) = ks, where k is constant. Which trajectory does the particle follow?

a) parabola
b) circle
c) ellipse
d) the data is not sufficient

Answer: d.

Task 3
Entry level (1 point)

How does the period of a mass-spring harmonic oscillator change if the amplitude of the oscillations is doubled?

a) decreases twofold
b) increases twofold
c) stays the same

d) increases by a factor of √2

Answer: c.

Task 4
Intermediate level (3 points)

A bullet of a mass of m = 40 gram is fired with a speed of v =1000 m/s into a ballistic gelatin sample of a thickness of d = 10 cm and goes through the sample. The drag force acting on the bullet depends upon the penetration depth x, given by $F(x)=F_0\cdot\exp(-x/d)$, where $F_0 = 10^5$ newton. Assume the bullet is moving along a straight line perpendicular to the surface of the sample. Determine the bullet's speed as it exits the sample and round the result to the nearest integer in m/s.

Answer: 827.

Task 5
Intermediate level (3 points)

A straight uniform bar of a length of L=0.5 m is suspended from a hinge O by one of its ends and oscillates in a vertical plane. The maximal speed of the bar's free end in the process of oscillations is v = 2.8 m/s. The acceleration of gravity is **g**=10 m/s^2. Find the maximum angle between the bar and the vertical. Round your answer to the nearest ten degrees.

Answer: 60.

Thermodynamics

Task 6
Entry level (1 point)

The density of solid Fe at melting temperature is 7.6 gr/cm^3, and of liquid Fe 7.4 gr/cm^3. How does the crystallization temperature change when the pressure increases from 1 bar (10^5 Pa) to 10 bar?

a) Melting is impossible at this condition.

b) It increases.

c) It decreases.

d) It does not change.

Answer: b.

Task 7
Entry level (1 point)

For a three-component system at an arbitrary but fixed pressure, which is the possible number of phases in equilibrium?

a) only one;
b) one or two;
c) any number;

d) not more than four.

Answer: d.

Task 8

Entry level (1 point)

Which of the following is true about the Gibbs energy function?

a) It increases with temperature, while the rate of increase decreases.

b) It decreases with a rise in both temperature and pressure.

c) It increases with a rise in both temperature and pressure.

d) It decreases with a rise in temperature and increases with a rise in pressure.

Answer: d.

Task 9

Intermediate level (3 points)

According to the Maxwell distribution for molecule velocity, the probability is $F(v) = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 \exp\left\{\frac{-mv^2}{2kT}\right\}$. What is the most probable velocity of deuterium molecules at a low pressure and a temperature of 600 K? Provide your answer in m/s, rounded to the nearest hundred.

Answer: 1600.

Task 10

Intermediate level (3 points)

Calculate the distance between the molecules of oxygen at a temperature of 1000 K and a pressure of $8.31 \cdot 10^{-5}$ Pa. The answer must be given in microns (1 micron = 10^{-6} meters), rounded to the first decimal place.

Answer: 5.5.

Engineering, electrical and electronic

Task 11

Entry level (1 point)

The center of a uniformly charged ball is located at a large distance, compared to the size of the ball, from an infinite uniformly charged plane. How does the force of interaction between the ball and the plane change if this distance is increased by a factor of 3?

a) It increases threefold.

b) It decreases threefold.

c) It decreases ninefold.

d) It does not change.

Answer: d.

Task 12
Entry level (1 point)

The area of the plates of a flat capacitor is halved, and the distance between them is increased threefold, while maintaining a constant voltage. How does the magnitude of the electrostatic interaction between the plates change?

a) It decreases by a factor of 6.
b) It decreases by a factor of 18.
c) It decreases by a factor of 12.
d) It increases by a factor of 1.5.

Answer: b.

Task 13
Entry level (1 point)

Two current-carrying circuits are located at a distance much greater than their size. If this distance is doubled, by how many times does the strength of their interaction decrease?

a) 2
b) 4
c) 8
d) 16

Answer: d.

Task 14
Intermediate level (3 points)

Inside a long solenoid with a circular cross-section and a constant current, there is a core with a circular cross-section of the same length, coaxial with the solenoid. The radius of the solenoid is 4 times the radius of the core, and the relative magnetic permeability of the core is 1500. By how many times is the magnetic energy in the core greater than the magnetic energy outside of it? Neglect edge effects. Provide your answer as an integer.

Answer: 100.

Task 15
Intermediate level (3 points)

A dielectric with a relative permittivity of 5 is introduced between the plates of an empty flat capacitor, filling one-third of the distance between the plates. By how many times does the energy of the capacitor change? The voltage between the plates remains constant. Provide your answer as a fraction.

Answer: 15/11.

Task 16
Advanced level (15 points)

Two identical empty flat capacitors are half-filled with a dielectric. In the first capacitor, the dielectric interface is perpendicular to the plates, while in the second capacitor, the interface is parallel to the plates. The capacitance of the first capacitor is 1.8 times greater than that of the second. Determine the relative permittivity of the dielectric. Neglect edge effects.
Please note that the evaluation will consider your problem-solving process; providing only the final answer is not sufficient.

Solution. Let us denote the capacitance of an empty capacitor by C_0. The first capacitor can be considered as two parallel connected capacitors, whose capacitance is $C_0/2$ and $\varepsilon C_0/2$ respectively **(worth 2 points)**. Then the capacity of the first capacitor is

$$C_1 = \frac{C_0}{2} + \frac{\varepsilon C_0}{2} = \frac{C_0}{2}(\varepsilon + 1)$$ **(worth 2 points)**.

(3.1.1)

The second capacitor can be represented as two capacitors connected in series, their capacitance equal to $2C_0$ and $2\varepsilon C_0$ respectively **(worth 2 points)**. Then the capacity of the second capacitor can be found using the following ratio:

$$\frac{1}{C_2} = \frac{1}{2C_0} + \frac{1}{2\varepsilon C_0} = \frac{\varepsilon+1}{2\varepsilon C_0}$$ **(worth 2 points)**.

(3.1.2)

Multiplying these two equations, we obtain

$$\frac{C_1}{C_2} = \frac{(\varepsilon+1)^2}{4\varepsilon} = 1.8$$ **(worth 4 points)**,

(3.1.3)

By solving this quadratic equation, we obtain $\varepsilon = 5$ **(3 points)**.

Optics

Task 17
Entry level (1 point)

The equation of a wave is given by: $\xi(x,t) = A\cos(1000 \cdot t - 250 \cdot x)$. Time is expressed in seconds and distance in meters. What is the speed of the wave?

a) 0.4 m/s
b) 0.25 m/s
c) 4 m/s
d) 250 km/s

Answer: c.

Task 18
Entry level (1 point)

A stationary source emits sound waves of a frequency of $\nu_0 = 1000\text{Hz}$. The receiver moves in the air at a speed of $v = 0.1v_S$ in the direction of the source, where v_S is the speed of sound in air. The sound of which frequency does the receiver register?

a) 1100 Hz
b) 900 Hz

c) 100 Hz
d) 909 Hz

Answer: a.

Task 19
Entry level (1 point)

How does the cutoff wavelength of bremsstrahlung X-ray radiation change when the accelerating voltage on Coolidge X-ray tube increases by a factor of 2?

a) It decreases by a factor of 2.
b) It increases by a factor of 2.
c) It decreases by a factor of 4.
d) It increases by a factor of 4.

Answer: a.

Task 20
Intermediate level (3 points)

A plane monochromatic light wave with a wavelength of λ=600 nm falls perpendicularly on an opaque screen with a circular aperture with a radius of 1.2 mm. The observation point is located on the axis of the aperture at a distance of b=2.0 mm from it. Determine the number of open Fresnel zones (Fresnel number) at the observation point. Round your answer to the nearest tenth.

Answer: 1.2.

Task 21
Intermediate level (3 points)

Find the power of thermal radiation emitted by a tungsten filament heated to 2000°C with a length of 1.0 cm and a diameter of 1.0 mm. Assume the filament to be a black body and the Stefan-Boltzmann constant equal to $\sigma = 5.67 \cdot 10^{-8} \mathrm{W} \cdot \mathrm{m}^{-2} \cdot \mathrm{K}^{-4}$. Write down the answer in watts (W), accurate to three significant digits.

Answer: 28.5.

Atomic, Molecular and Chemical Physics

Task 22
Entry level (1 point)

Find the de Broglie wavelength for electrons accelerated by a potential difference of 70 V. The rest energy of the electron is 0.511 MeV. Use Planck's constant $\hbar = 6.58 \cdot 10^{-16}$ eV·s and the speed of light $c = 3 \cdot 10^{8}$ m/s. Provide your answer in nm, rounded to the nearest hundredth.

a) 1.29
b) 0.49
c) 0.15

d) 1.49

Answer: c.

Task 23
Entry level (1 point)

Find the Compton wavelength of a positron whose rest energy is 0.511 MeV. Use Planck's constant $\hbar = 1.05 \cdot 10^{-34}$ J·s and the speed of light $c = 3 \cdot 10^8$ m/s. Provide your answer in pm, rounded to the nearest hundredth.

a) 0.39
b) 1.39
c) 2.58
d) 0.00

Answer: a.

Task 24
Entry level (1 point)

Calculate the change in the orbital magnetic moment of an electron during the transition of a hydrogen atom from the ground state to the 3d state. Provide your answer in Bohr magnetons, rounded to the nearest tenth.

a) 1.4
b) 2.4
c) 3.4
d) 0.0

Answer: b.

Task 25
Intermediate level (3 points)

Determine the speed a hydrogen atom gains after emitting a photon during the transition from the third energy level (n=3) to the first. The speed of light is $c = 3 \cdot 10^8$ m/s, the mass of the proton mass is 938 MeV. Provide your answer in cm/s, rounded to the nearest tenth.

Answer: 3.9.

Physics of Condensed Matter

Task 26
Entry level (1 point)

The mobility of the electron in a metal is 100 $cm^2/(V \cdot s)$. How far does an electron travel in 1 ms when a potential difference of 0.5 V is applied to a conductor that is 2 m long?

a) 2.5 cm
b) $2.5 \cdot 10^{-4}$ m
c) $2.5 \cdot 10^{-4}$ cm
d) 2.5 cm

Answer: c.

Task 27
Entry level (1 point)

Which type of defect is exemplified by dislocation?

a) equilibrium point-like defect
b) equilibrium linear defect
c) non-equilibrium point-like defect
d) non-equilibrium linear defect

Answer: d.

Task 28
Entry level (1 point)

The band gap width in a certain substance is 1.38 eV. What wavelength of radiation from an external source would be sufficient to increase electroconductivity?

a) 0.9 micron
b) 0.5 micron
c) 0.1 micron
d) 3 micron

Answer: a.

Task 29
Intermediate level (3 points)

Silver (Ag) is a FCC lattice metal (lattice constant a= 0.409 nm) with an atomic mass of M=107.9 g/mol. Calculate the theoretical density of silver. Calculate the theoretical density of silver, providing the answer to the nearest hundred kg/m^3.

Answer: 10500.

Task 30
Advanced level (15 points)

Silicon (Si) is doped with boron (B) at a concentration of 10^{16} at/cm^3. Estimate the conductivity of the silicon at a temperature of 400 K, given that the mobility of electrons (l_e) at this temperature is 1000 $cm^2/(V \cdot s)$, the mobility of holes (l_p) is 100 $cm^2/(V \cdot s)$, the conductivity of pure Si at 300 K is $5 \cdot 10^{-5}$ Ohm^{-1} cm^{-1}, and the band gap (E_F) for Si is 1.1 eV. Provide the answer in Ohm^{-1} cm^{-1}, rounded to one significant figure.

Please note that the evaluation will consider your problem-solving process; providing only the final answer is not sufficient.

Solution:

Step 1. Doping with boron (B) results in p-type conductivity, where each boron atom creates one hole. Thus the hole conductivity is estimated as follows:

$\sigma = e * N_p * l_p$=0.16 **Om^{-1} cm^{-1}**

Step 2. The intrinsic conductivity of Si can be obtained from the temperature dependence of conductivity with known band gap as $\ln\left(\frac{\sigma(T_2)}{\sigma(T_1)}\right) = \frac{E_F}{2k}\left\{\frac{1}{T_2} - \frac{1}{T_1}\right\}$

σ(400K)=0.00102 **Om^{-1} cm^{-1}.**

Step 3. If the intrinsic conductivity is much lower than the hole conductivity, the total conductivity is 0.16 **Om^{-1} cm^{-1} .**

With the rounding we get: 0.2

Criteria:

Estimating the intrinsic conductivity is worth 5 points.

Defining the conductivity type is worth 5 points.

Calculating the total conductivity and formulating the answer is worth 5 points.

Quantum Technologies

Task 31
Entry level (1 point)

Which is the mean value of the momentum of a particle in a state with the wave function

$$\psi(\vec{\mathbf{r}}) = \frac{\sqrt{2}i}{(2\pi\hbar)^{3/2}}\sin(\vec{\mathbf{k}}\vec{\mathbf{r}})$$

.

a) $\hbar k$
b) 0
c) $2\hbar k$
d) $\hbar k / 2$

Answer: b.

Task 32
Entry level (1 point)

An ideal Fermi-gas consisting of N particles is in equilibrium at the temperature T, its chemical potential is μ. Select the mean number of particles on the single-particle energy level ε, given that the degeneracy of the level is α.

a) 1
b) α

c) $\dfrac{\alpha}{\exp\left(\dfrac{\varepsilon-\mu}{k_B T}\right)+1}$

d) $\dfrac{1}{\exp\left(\dfrac{\varepsilon-\mu}{k_B T}\right)+1}$

Answer: c.

Task 33
Entry level (1 point)

The wave function of a particle has the form $\psi(\vec{\mathbf{r}}) = \dfrac{1}{\sqrt{\pi a^3}} \exp\left(-\dfrac{|\vec{\mathbf{r}}|}{a}\right)$, where α is a given constant. Select the most probable value of $|\vec{\mathbf{r}}|$.

a) a
b) $3/2a$
c) $\sqrt{3}a$
d) $\sqrt{21}/2a$

Answer: a.

Task 34
Intermediate level (worth 4 points)

At some instant, the normalized wave function of a system has the form $\psi(q) = \dfrac{1}{\sqrt{3}}\psi_{A=3}(q) + \sqrt{\dfrac{2}{3}}\psi_{A=6}(q)$, where $\psi_{A=3}(q)$ and $\psi_{A=6}(q)$ are the normalized eigen-functions of the operator of the physical quantity A, corresponding to the eigenvalues A=3 and A=6, respectively. What is the mean value of A at this instant? Provide the answer as an integer.
Answer: 5

Task 35
Advanced level (15 points)

A uniform electric field is suddenly applied to a charged oscillator in its ground state. Determine the probabilities of oscillator transitioning to excited states as a result of this perturbation. The mass and frequency of the oscillator are m and ω, respectively. The electric field force acting on the oscillator is F.
Please note that the evaluation will consider your problem-solving process; providing only the final answer is not sufficient.

Solution:
The potential energy of the oscillator in the uniform field is

$$U(x)=\frac{m\omega^2x^2}{2}-Fx=$$
$$=\frac{m\omega^2}{2}\left(x^2-2\frac{F}{m\omega^2}x+\left(\frac{F}{m\omega^2}\right)^2-\left(\frac{F}{m\omega^2}\right)^2\right)=$$
$$=\frac{m\omega^2}{2}(x-x_0)^2+const \qquad (3.3.1)$$

(where $x_0=F/(m\omega^2)$), i.e. it still has the pure oscillator form despite the shift in the equilibrium position. Hence, the wave functions of the stationary states of the perturbed oscillator are $\psi_n(x-x_0)$, where $\psi_n(x)$ are the wave-functions of the stationary states of the unperturbed oscillator.

(worth 5 points)

The wave function of the perturbed oscillator can be expanded by the normalized wave functions of the unperturbed oscillator

$$\psi_n(x-x_0)=\sum_k C_{k,n}\psi_k(x-x_0), \qquad (3.3.2)$$

where

$$C_{k,n}=\int dx\psi_k^*(x)\psi_n(x-x_0). \qquad (3.3.3)$$

The square of the modulus of the expansion coefficient $C_{k,n}$ gives the probability of transition of the oscillator from state k to state n. Thus, the probability of transition from the ground state (k=0) to the nth excited state is

$$P_n=\left|\int dx\psi_0(x)\psi_k(x-x_0)\right|^2. \qquad (3.3.4)$$

Substituting the explicit form of the wave function of the stationary state of the oscillator yields:

$$\psi_n(x)=\frac{1}{\sqrt{2^n n!\sqrt{\pi}\ell}}\exp\left(-\frac{x^2}{2\ell^2}\right)H_n\left(\frac{x}{\ell}\right), \qquad (3.3.5)$$

Thus, we obtain

$$P_n=\frac{1}{2^n n!\pi}\left|\int d\left(\frac{x}{\ell}\right)\exp\left(-\frac{x^2}{2\ell^2}-\frac{(x-x_0)^2}{2\ell^2}\right)H_n\left(\frac{x-x_0}{\ell}\right)\right|^2,. \qquad (3.3.6)$$

where $\ell=\sqrt{\hbar/(m\omega)}$

(worth 5 points)

After deriving the full square

$$-\frac{x^2}{2\ell^2}-\frac{(x-x_0)^2}{2\ell^2}=-\left(\frac{x}{\ell}-\frac{x_0}{2\ell}\right)^2-\left(\frac{x_0}{2\ell}\right)^2 \qquad (3.3.7)$$

and introducing the new variable $\xi \equiv \frac{x}{\ell} - \frac{x_0}{2\ell}$, we obtain

$$P_n = \frac{1}{2^n n! \pi} \exp\left(-2\left(\frac{x_0}{2\ell}\right)^2\right) \left| \int d\xi \exp\left(-\xi^2\right) H_n\left(\xi - \frac{x_0}{2\ell}\right) \right|^2 ,. \qquad (3.3.8)$$

Using the expression for Hermite polynomials $H_n(x)$

$$H_n(y+x) = \sum_{k=0}^{n} \binom{n}{k} H_{n-k}(x)(2y)^k , \qquad (3.3.9)$$

we get

$$\int d\xi \exp\left(-\xi^2\right) H_n\left(\xi - \frac{x_0}{2\ell}\right) = \sum_{k=0}^{n} (-1)^k \binom{n}{k} \left(\frac{x_0}{\ell}\right)^k \int d\xi \exp\left(-\xi^2\right) H_{n-k}(\xi) =$$

$$= \sum_{k=0}^{n} (-1)^k \binom{n}{k} \left(\frac{x_0}{\ell}\right)^k \int d\xi \exp\left(-\xi^2\right) H_0(\xi) H_{n-k}(\xi) =$$

$$= \sum_{k=0}^{n} (-1)^k \binom{n}{k} \left(\frac{x_0}{\ell}\right)^k \delta_{n-k,0} 2^0 0! \sqrt{\pi} = (-1)^n \sqrt{\pi} \left(\frac{x_0}{\ell}\right)^n \qquad (3.3.10)$$

Thus, we obtain the Poisson distribution

$$P_n = \frac{\langle n \rangle^n}{n!} \exp\left(-\langle n \rangle\right) \qquad (3.3.11)$$

with average value $\langle n \rangle = \sum_{n=0}^{+\infty} n P_n = \left(\frac{x_0}{\sqrt{2}\ell}\right)^2 = \frac{F^2}{2m\hbar\omega^3}$.

(worth 5 points)

Answer: $P_n = \frac{\langle n \rangle^n}{n!} \exp\left(-\langle n \rangle\right)$, where $\langle n \rangle = \frac{F^2}{2m\hbar\omega^3}$

Assessment criteria:

1) Obtaining the wave functions of the stationary states of an oscillator in a homogeneous field is worth 5 points.

2) Calculating the general expression for the probability of the required transitions is worth 5 point.

3) Performing integration and obtaining the final expression (3.3.11) is worth 5 points.