Международная олимпиада Ассоциации «Глобальные университеты»
для абитуриентов магистратуры (МОАМ)

Программа по профилю «Математика»

1. Limit of a sequence and its properties. Cauchy criterion. Limit points of a sequence, upper and lower limits. Bolzano–Weierstrass theorem.
2. Limit of a function of one variable at a given point and its properties. Two definitions of limit of a function (Heine and Cauchy) and their equivalence. Cauchy criterion.
3. Continuous functions of one variable. Properties of a continuous function on a segment (Weierstrass and Bolzano–Cauchy theorems). Inverse function theorem. Uniform continuity. Cantor’s theorem.
4. Derivative of functions of one variable and its properties. Chain rule. Differentiable functions. Differential of a function. Derivative of an inverse function.
5. Derivatives and differentials of higher orders. General Leibniz rule.
6. Rolle’s theorem, mean value theorem, Cauchy’s mean value theorem for differentiable functions. L’Hôpital’s rule.
7. Taylor’s theorem. Peano’s and Lagrange’s form of the remainder. Evaluating limits of functions using Taylor’s theorem and L’Hôpital’s rule.
8. Exploring functions of one variable using derivatives (monotonicity, maxima and minima, convexity, inflection points).
9. Differentiable functions of several variables. Necessary conditions. Sufficient conditions. Gradient of a function.
10. Implicit function theorem.
11. Maxima and minima of a function of several variables. Necessary conditions. Sufficient conditions.
12. Maxima and minima of a function subject to equality constraints. Lagrange multipliers method. Necessary conditions. Sufficient conditions.
13. Definite (Riemann) integral of a function of one variable. Darboux integrals. Darboux criterion. Fundamental theorem of calculus. Newton–Leibniz’s axiom. Geometric applications of definite integrals.
14. Improper integrals. Absolute and conditional convergence. Cauchy criterion. Convergion tests for improper integrals (comparison test, Dirichlet’s test).
15. Numerical series. Absolute and conditional convergence. Cauchy criterion. Convergion tests (comparison tests, integral test, root test, ratio test, Dirichlet’s test).
16. Functional series. Uniform convergence. Cauchy criterion. Weierstrass M-test and Dirichlet’s test of uniform convergence.
17. Power series. Radius of convergence, Cauchy–Hadamard theorem. Taylor series. Taylor series for elementary functions.
18. Line integrals. Green’s theorem.
19. Surface integrals. Divergence theorem and Stokes’ theorem.
20. Riemann–Lebesgue lemma. Trigonometric Fourier series. Conditions for convergence at a point. Conditions of uniform convergence on a segment.
21. Fourier transform for absolutely integrable functions and its properties. Fourier transform of derivative and derivative of Fourier transform.
22. Vector algebra. Cross product, dot product and triple product.
23. Different types of equations of line and plane. Angles between planes and lines. Distance from a point to a line, distance from a point to a plane. Distance between skew lines.
24. Second-order curves. Ellipse, hyperbola, parabola and their properties. Second-order surfaces. Ellipsoid, hyperboloid of one sheet, hyperboloid of two sheets, elliptic paraboloid, hyperbolic paraboloid, cone.

25. Affine transformations of a plane and its properties. Sign and absolute of the determinant of the transformation matrix and their geometric sense.
26. Orthogonal transformations of a plane and their properties.
27. Systems of linear equations. Cramer's rule. Rouché–Capelli theorem. Fredgolm's theorem. General solution of a system of linear equations.
28. Linear space, its basis and dimension. Subspaces, sum and intersection of subspaces. Linear maps of finite-dimensional linear spaces, their matrices. Changes in a linear map matrix when changing the basis. Eigenvalues, eigenvectors and their properties.
29. Quadratic functions and their canonical form.
30. Finite-dimensional Euclidian spaces. Gram matrix. Adjoint linear transformations and their properties.
31. Self-adjoint linear transformations of finite-dimensional Euclidian spaces. Properties of their eigenvectors and eigenvalues.
32. Ordinary differential equations. Separating the variables. Reduction of order of differential equations. Using parameter for solving first-order implicit ordinary differential equations.
33. Linear ordinary differential equations and systems of linear differential equations with constant coefficients.
34. Linear ordinary differential equations and systems of linear differential equations with variable coefficients. Fundamental set of solutions. Wronskian. Liouville's formula. Variation of parameters.
35. Calculus of variations. Euler–Lagrange equation. Necessary conditions of maxima and minima.
36. Equilibrium points of autonomous systems of ordinary differential equations. Types of equilibria for two-dimensional systems. Lyapunov stability.
37. First integrals of autonomous systems of differential equations. Number of functionally independent first integrals. Linear partial differential equations of the first order, general solution and Cauchy's problem.
38. Probability space. Independent events. Conditional probability. Exhaustive events. Law of total probability. Bayes' rule.
39. Random variable and its cumulative distribution function. Expected value and variance of a random variable.
40. Main distribution types of random variables: binomial, geometric, uniform, Poisson, exponential, Gaussian.
41. Bernoulli trial. Chebyshev's inequality and law of large numbers.
42. Joint probability distribution of several random variables. Independence of random variables. Covariation. Correlation coefficient.
43. Complex numbers and their properties.
44. Regular functions of one complex variable. Cauchy's integral formula.
45. Calculating residues. Evaluating integrals using residues. Jordan's lemma.
46. Conformal maps. Möbius transformation and its properties. Jukowsky transform and its properties.