Physical Sciences and Technology: Second-round Sample Tasks for the Open Doors Master's and Doctoral Track

This sample test comprises 35 tasks, including: 21 entry-level tasks with a single correct answer, each correct answer assigned 1 point; 11 intermediate-level tasks with multiple correct answers, each correct answer is assigned 3 to 4 points; 3 advanced-level tasks requiring a detailed answer, each answer assigned up to 15 points, depending on its correctness and completeness.
For advanced-level tasks requiring a detailed answer, assessment criteria and a standard answer are provided.

Field of Science 1. Mechanics

Task 1
Entry level (1 point)

An object with a mass of 100 kg slides down an inclined plane at a constant velocity. The plane is inclined at an angle of 30°. What is the magnitude of the total force acting on the object? The answer options are given in kilonewtons.

a) 0
b) 1
c) 2
d) 4

Answer: a.

Task 2
Entry level (1 point)

The magnitude of the total force acting on a particle depends on the displacement s, according to the equation $F(s) = ks$, where k is a constant. What trajectory does the particle follow?

a) parabola
b) circle
c) ellipse
d) the provided data are not sufficient

Answer: d.

Task 3
Entry level (1 point)

How does the period of a mass-spring harmonic oscillator change when the oscillation amplitude is doubled?

a) It decreases twofold.
b) It increases twofold.
c) It remains unchanged.
d) It increases by a factor of $\sqrt{2}$.

Answer: c.

Task 4
Intermediate level (3 points)

A bullet of mass m = 40 grams is fired with a speed of v = 1000 m/s into a ballistic gelatin sample of thickness d = 10 cm, passing through the sample. The drag force acting on the bullet depends on the penetration depth x, given by $F(x) = F_0 \cdot \exp(-x/d)$, where $F_0 = 10^5$ newtons. Assume the bullet moves along a straight line perpendicular to the surface of the sample. Determine the bullet's speed as it exits the sample and round the result to the nearest integer in m/s.
Answer: 827.

Task 5
Intermediate level (3 points)

A straight uniform bar of a length L = 0.5 m is suspended from a hinge O by one end and oscillates in a vertical plane. The maximum speed of the bar's free end during oscillations is v = 2.8 m/s. The acceleration due to gravity is g = 10 m/s². Find the maximum angle between the bar and the vertical. Round your answer to the nearest ten degrees.
Answer: 60.

Field of Science 2. Thermodynamics

Task 6
Entry level (1 point)

The density of solid Fe at melting temperature is 7.6 g/cm³, and that of liquid Fe is 7.4 g/cm³. How does the crystallization temperature change when the pressure increases from 1 bar (10^5 Pa) to 10 bar?
a) Melting is impossible under this condition.
b) It increases.
c) It decreases.
d) It remains unchanged.
Answer: b.

Task 7
Entry level (1 point)

What is the possible number of phases in equilibrium in a three-component system at a fixed pressure?

a) only one
b) one or two
c) any number
d) no more than four.

Answer: d.

Task 8
Entry level (1 point)

Which of the following is true of the Gibbs energy function?

a) It increases with temperature, while the rate of increase declines.
b) It decreases with a rise in both temperature and pressure.
c) It increases with a rise in both temperature and pressure.
d) It decreases with a rise in temperature and increases with a rise in pressure.

Answer: d.

Task 9
Intermediate level (3 points)

According to the Maxwell distribution for molecule velocity, $F(v) = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 \exp\left\{\frac{-mv^2}{2kT}\right\}$. What is the most probable velocity of deuterium molecules at low pressure and a temperature of 600 K? Provide your answer in m/s, rounded to the nearest hundred.
Answer: 1600.

Task 10
Intermediate level (3 points)

Calculate the distance between oxygen molecules at a temperature of 1000 K and a pressure of 8.31×10^{-5} Pa. Provide your answer in microns (1 micron = 10^{-6} meters), rounded to the first decimal place.
Answer: 5.5.

Field of Science 3. Electrical and Electronics Engineering

Task 11
Entry level (1 point)

The center of a uniformly charged ball is located at a large distance from an infinite uniformly charged plane compared to the size of the ball. How does the force of interaction between the ball and the plane change if this distance increases by a factor of 3?
- a) It increases threefold.
- b) It decreases threefold.
- c) It decreases ninefold.
- d) It remains unchanged.

Answer: d.

Task 12
Entry level (1 point)

How does the magnitude of the electrostatic interaction between the plates of a flat capacitor change if the area of the plates is halved and the distance between them is tripled while maintaining constant voltage?
- a) It decreases by a factor of 6.
- b) It decreases by a factor of 18.
- c) It decreases by a factor of 12.
- d) It increases by a factor of 1.5.

Answer: b.

Task 13
Entry level (1 point)

If two current-carrying circuits are separated by a distance much greater than their size, by what factor does the strength of their interaction decrease when this distance is doubled?
- a) 2

b) 4
c) 8
d) 16

Answer: d.

Task 14
Intermediate level (3 points)

A core with a circular cross-section and the same length is placed inside a long solenoid with a circular cross-section and constant current, coaxial with the solenoid. The radius of the solenoid is 4 times the radius of the core, and the relative magnetic permeability of the core is 1500. By what factor is the magnetic energy within the core greater than the magnetic energy outside it? Neglect edge effects. Provide your answer as an integer.

Answer: 100.

Task 15
Intermediate level (3 points)

A dielectric with a relative permittivity of 5 is introduced between the plates of an empty flat capacitor, filling one-third of the distance between the plates. By what factor does the energy of the capacitor change? The voltage between the plates remains constant. Provide your answer as a fraction.

Answer: 15/11.

Task 16
Advanced level (15 points)

Two identical empty flat capacitors are half-filled with a dielectric. In the first, the dielectric boundary is perpendicular to the plates; in the second, it is parallel. The capacitance of the first is 1.8 times that of the second. What is the relative permittivity of the dielectric? Neglect edge effects.

Note: A complete solution must include your method and reasoning. Providing the final answer alone will not suffice.

Solution. Let the capacitance of the empty capacitor be C_0. The first capacitor can be modeled as two capacitors connected in parallel, with capacitances $C_1 = C_0$ and $C_2 = \varepsilon \cdot C_0$, respectively (**2 points**). Therefore, the total capacitance of the first capacitor is

$$C_1 = \frac{C_0}{2} + \frac{\varepsilon C_0}{2} = \frac{C_0}{2}(\varepsilon + 1) \quad \textbf{(2 points)}. \tag{3.1.1}$$

The second capacitor can be represented as two capacitors connected in series, with capacitances $2C_0$ and $2\varepsilon C_0$ respectively (**2 points**).

Then the total capacitance of the second capacitor is given by the expression:

$$\frac{1}{C_2} = \frac{1}{2C_0} + \frac{1}{2\varepsilon C_0} = \frac{\varepsilon + 1}{2\varepsilon C_0} \quad \textbf{(2 points)}. \tag{3.1.2}$$

Multiplying these two equations yelids

$$\frac{C_1}{C_2} = \frac{(\varepsilon+1)^2}{4\varepsilon} = 1.8 \quad \textbf{(4 points)}, \tag{3.1.3}$$

By solving this quadratic equation, we obtain $\varepsilon = 5$ (**3 points**).

Field of Science 4. Optics

Task 17
Entry level (1 point)

The equation of a wave is given by: $\xi(x, t) = A\cos(1000 \cdot t - 250 \cdot x)$. Time is expressed in seconds and distance in meters. What is the speed of the wave?

a) 0.4 m/s
b) 0.25 m/s
c) 4 m/s
d) 250 km/s

Answer: c.

Task 18
Entry level (1 point)

A stationary source emits sound waves at a frequency of $\nu_0 = 1000\text{Hz}$. The receiver moves in the air at a speed of $v = 0.1v_S$ in the direction of the source, where v_S is the speed of sound in air. The sound of what frequency does the receiver register?

a) 1100 Hz
b) 900 Hz
c) 100 Hz
d) 909 Hz

Answer: a.

Task 19
Entry level (1 point)

By what factor does the cutoff wavelength of bremsstrahlung X-ray radiation change if the accelerating voltage in a Coolidge X-ray tube is doubled?

a) It decreases by a factor of 2.
b) It increases by a factor of 2.
c) It decreases by a factor of 4.
d) It increases by a factor of 4.

Answer: a.

Task 20
Intermediate level (3 points)

A plane monochromatic light wave with a wavelength of $\lambda = 600$ nm falls perpendicularly on an opaque screen with a circular aperture of radius 1.2 mm. The observation point is located on the axis of the aperture at a distance of b = 2.0 mm. Determine the number of open Fresnel zones (Fresnel number) at the observation point. Round your answer to the nearest tenth.

Answer: 1.2.

Task 21
Intermediate level (3 points)

Find the power of thermal radiation emitted by a tungsten filament heated to 2000°C with a length of 1.0 cm and a diameter of 1.0 mm. Assume the filament to be a black body and the Stefan-Boltzmann constant to be $\sigma = 5.67 \cdot 10^{-8} \text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$. Write down the answer in watts (W), accurate to three significant digits.

Answer: 28.5.

Field of Science 5. Atomic, Molecular, and Chemical Physics

Task 22
Entry level (1 point)

Find the de Broglie wavelength for electrons accelerated by a potential difference of 70 V. The rest energy of the electron is 0.511 MeV. Use Planck's constant $\hbar = 6.58 \cdot 10^{-16}$ eV·s and the speed of light $c = 3 \cdot 10^{8}$ m/s. Provide your answer in nm, rounded to the nearest hundredth.

a) 1.29
b) 0.49
c) 0.15
d) 1.49

Answer: c.

Task 23
Entry level (1 point)

Find the Compton wavelength of a positron whose rest energy is 0.511 MeV. Use Planck's constant $\hbar = 1.05 \cdot 10^{-34}$ J·s and the speed of light $c = 3 \cdot 10^{8}$ m/s. Provide your answer in pm, rounded to the nearest hundredth.

a) 0.39
b) 1.39
c) 2.58
d) 0.00

Answer: a.

Task 24
Entry level (1 point)

Calculate the change in the orbital magnetic moment of an electron during the transition of a hydrogen atom from the ground state to the 3d state. Provide your answer in Bohr magnetons, rounded to the nearest tenth.

a) 1.4
b) 2.4
c) 3.4
d) 0.0

Answer: b.

Task 25
Intermediate level (3 points)

Determine the speed gained by a hydrogen atom after emitting a photon during the transition from the third energy level (n = 3) to the first (n = 1). The speed of light is $c = 3 \cdot 10^{8}$ m/s, the mass of the proton mass is 938 MeV. Provide your answer in cm/s, rounded to the nearest tenth.

Answer: 3.9.

Field of Science 6. Physics of Condensed Matter

Task 26
Entry level (1 point)

The mobility of an electron mobility in a metal is 100 cm²/(V·s). How far does the electron travel in 1 ms when a potential difference of 0.5 V is applied across a 2 m long conductor?

a) 2.5 cm
b) $2.5 \cdot 10^{-4}$ m
c) $2.5 \cdot 10^{-4}$ cm
d) 2.5 cm

Answer: c.

Task 27
Entry level (1 point)

Which type of defect is exemplified by dislocation?

a) equilibrium point defect
b) equilibrium line defect
c) non-equilibrium point defect
d) non-equilibrium line defect

Answer: d.

Task 28
Entry level (1 point)

The band gap width in a substance is 1.38 eV. What wavelength of radiation from an external source is sufficient to increase electroconductivity?

a) 0.9 micron
b) 0.5 micron
c) 0.1 micron
d) 3 micron

Answer: a.

Task 29
Intermediate level (3 points)

Silver (Ag) is a FCC lattice metal (lattice constant a = 0.409 nm) with an atomic mass of M=107.9 g/mol. Calculate the theoretical density of silver, rounding y answer to the nearest hundred kg/m³.

Answer: 10500.

Task 30
Advanced level (15 points)

Silicon (Si) is doped with boron (B) at a concentration of 10^{16} at/cm^3. Estimate the conductivity of the silicon at a temperature of 400 K, given that the mobility of electrons (l_e) at this temperature is 1000 cm^2/(V·s), the mobility of holes (l_p) is 100 cm^2/(V·s), the conductivity of pure Si at 300 K is $5 \cdot 10^{-5}$ Ohm^{-1} cm^{-1}, and the band gap (E_F) for Si is 1.1 eV. Provide the answer in Ohm^{-1} cm^{-1}, rounded to one significant figure.

Note: A complete solution must include your method and reasoning. Providing the final answer alone will not suffice.

Solution:
Step 1. Doping with boron (B) results in p-type conductivity, where each boron atom creates one hole. Thus the hole conductivity is estimated as follows:
$\sigma = e * N_p * l_p$=0.16 **Om⁻¹ cm⁻¹**
Step 2. The intrinsic conductivity of Si can be obtained from the temperature dependence of conductivity with known band gap as $\ln\left(\frac{\sigma(T_2)}{\sigma(T_1)}\right) = \frac{E_F}{2k}\left\{\frac{1}{T_2} - \frac{1}{T_1}\right\}$
σ(400K)=0.00102 **Om^{-1} cm^{-1}**.
Step 3. If the intrinsic conductivity is much lower than the hole conductivity, the total conductivity is 0.16 **Om^{-1} cm^{-1}** .
With rounding, the result is 0.2

Assessment criteria
Estimating the intrinsic conductivity is worth 5 points.
Defining the conductivity type earns 5 points.
Calculating the total conductivity and formulating the answer is worth 5 points.

Field of Science 7. Quantum Technologies

Task 31
Entry level (1 point)

What is the mean value of the momentum of a particle in a state with the wave function

$$\psi(\vec{\mathbf{r}}) = \frac{\sqrt{2}i}{(2\pi\hbar)^{3/2}}\sin\left(\vec{\mathbf{k}}\vec{\mathbf{r}}\right)$$
.

a)
b) 0
c) $2\hbar k$
d) $\hbar k / 2$

Answer: b.

Task 32
Entry level (1 point)

An ideal Fermi gas consisting of N particles is in equilibrium at temperature T, its chemical potential is μ. Identify the mean number of particles on the single-particle energy level ε given that the degeneracy of the level is α.

a) 1
b) α
c) $\dfrac{\alpha}{\exp\left(\dfrac{\varepsilon-\mu}{k_B T}\right)+1}$

d) $\dfrac{1}{\exp\left(\dfrac{\varepsilon-\mu}{k_B T}\right)+1}$

Answer: c.

Task 33
Entry level (1 point)

The wave function of a particle has the form $\psi(\vec{\mathbf{r}})=\dfrac{1}{\sqrt{\pi a^3}}\exp\left(-\dfrac{|\vec{\mathbf{r}}|}{a}\right)$, where α is a given constant. Determined the most probable value of $|\vec{\mathbf{r}}|$.

a) a
b) $3/2a$
c) $\sqrt{3}a$
d) $\sqrt{21}/2a$

Answer: a.

Task 34
Intermediate level (4 points)

At some instant, the normalized wave function of a system has the form $\psi(q)=\dfrac{1}{\sqrt{3}}\psi_{A=3}(q)+\sqrt{\dfrac{2}{3}}\psi_{A=6}(q)$, where $\psi_{A=3}(q)$ and $\psi_{A=6}(q)$ are the normalized eigen-functions of the operator of the physical quantity A, corresponding to the eigenvalues A = 3 and A = 6, respectively. What is the mean value of A at this instant? Provide the answer as an integer.

Answer: 5

Task 35
Advanced level (15 points)

A uniform electric field is suddenly applied to a charged oscillator in its ground state. Determine the probabilities of oscillator transitioning to excited states as a result of this perturbation. The mass and frequency of the oscillator are m and ω, respectively. The electric field force acting on the oscillator is F.

Note: A complete solution must include your method and reasoning. Providing the final answer alone will not suffice.

Solution:

The potential energy of the oscillator in the uniform field is

$$U(x)=\frac{m\omega^2x^2}{2}-Fx=$$
$$=\frac{m\omega^2}{2}\left(x^2-2\frac{F}{m\omega^2}x+\left(\frac{F}{m\omega^2}\right)^2-\left(\frac{F}{m\omega^2}\right)^2\right)=$$
$$=\frac{m\omega^2}{2}(x-x_0)^2+const \qquad (3.3.1)$$

(where $x_0=F/\left(m\omega^2\right)$), i.e. it still has the pure oscillator form despite the shift in the equilibrium position. Hence, the wave functions of the stationary states of the perturbed oscillator are $\psi_n(x-x_0)$, where $\psi_n(x)$ are the wave-functions of the stationary states of the unperturbed oscillator.

(5 points)

The wave function of the perturbed oscillator can be expanded in terms of the normalized wave functions of the unperturbed oscillator

$$\psi_n(x-x_0)=\sum_k C_{k,n}\psi_k(x-x_0), \qquad (3.3.2)$$

where

$$C_{k,n}=\int dx\psi_k^*(x)\psi_n(x-x_0). \qquad (3.3.3)$$

The square of the modulus of the expansion coefficient $C_{k,n}$ describes the probability of transition of the oscillator from state k to state n. Thus, the probability of transition from the ground state (k=0) to the nth excited state is

$$P_n=\left|\int dx\psi_0(x)\psi_k(x-x_0)\right|^2.$$

(3.3.4)

Substituting the explicit form of the wave function of the oscillator's stationary state yields:

$$\psi_n(x)=\frac{1}{\sqrt{2^n n!\sqrt{\pi}\ell}}\exp\left(-\frac{x^2}{2\ell^2}\right)H_n\left(\frac{x}{\ell}\right), \qquad (3.3.5)$$

Thus, we obtain

$$P_n=\frac{1}{2^n n!\pi}\left|\int d\left(\frac{x}{\ell}\right)\exp\left(-\frac{x^2}{2\ell^2}-\frac{(x-x_0)^2}{2\ell^2}\right)H_n\left(\frac{x-x_0}{\ell}\right)\right|^2,. \qquad (3.3.6)$$

where $\ell=\sqrt{\hbar/(m\omega)}$

(5 points)

After deriving the full square

$$-\frac{x^2}{2\ell^2}-\frac{(x-x_0)^2}{2\ell^2}=-\left(\frac{x}{\ell}-\frac{x_0}{2\ell}\right)^2-\left(\frac{x_0}{2\ell}\right)^2 \qquad (3.3.7)$$

and introducing the new variable $\xi\equiv\frac{x}{\ell}-\frac{x_0}{2\ell}$, we obtain

$$P_n = \frac{1}{2^n n!\pi}\exp\left(-2\left(\frac{x_0}{2\ell}\right)^2\right)\left|\int d\xi \exp\left(-\xi^2\right)H_n\left(\xi-\frac{x_0}{2\ell}\right)\right|^2,. \qquad (3.3.8)$$

Using the expression for Hermite polynomials $H_n(x)$ yields

, (3.3.9)

$$\int d\xi \exp\left(-\xi^2\right)H_n\left(\xi-\frac{x_0}{2\ell}\right) = \sum_{k=0}^{n}(-1)^k\binom{n}{k}\left(\frac{x_0}{\ell}\right)^k\int d\xi \exp\left(-\xi^2\right)H_{n-k}(\xi) =$$

$$= \sum_{k=0}^{n}(-1)^k\binom{n}{k}\left(\frac{x_0}{\ell}\right)^k\int d\xi \exp\left(-\xi^2\right)H_0(\xi)H_{n-k}(\xi) =$$

$$= \sum_{k=0}^{n}(-1)^k\binom{n}{k}\left(\frac{x_0}{\ell}\right)^k\delta_{n-k,0}2^0 0!\sqrt{\pi} = (-1)^n\sqrt{\pi}\left(\frac{x_0}{\ell}\right)^n \qquad (3.3.10)$$

Thus, we obtain the Poisson distribution

$$P_n = \frac{\langle n\rangle^n}{n!}\exp\left(-\langle n\rangle\right) \qquad (3.3.11)$$

with average value $\langle n\rangle = \sum_{n=0}^{+\infty} nP_n = \left(\frac{x_0}{\sqrt{2}\ell}\right)^2 = \frac{F^2}{2m\hbar\omega^3}$.

(5 points)

Answer: $P_n = \frac{\langle n\rangle^n}{n!}\exp\left(-\langle n\rangle\right)$, where $\langle n\rangle = \frac{F^2}{2m\hbar\omega^3}$

Assessment criteria

1) Obtaining the wave functions of the stationary states of an oscillator in a homogeneous field is worth 5 points.
2) Calculating the general expression for the probability of the required transitions earns 5 points.
3) Performing integration and obtaining the final expression (3.3.11) is worth 5 points.