# Master's, Doctoral, and Post-doctoral Track Program: Applied Mathematics and Artificial Intelligence

## 1. Open Doors winner's skill set

Winning the Open Doors competition requires:

- in-depth knowledge of Mathematics, Algorithm theory, Machine learning, and Information technology;
- practical skills in solving complex problems in data analysis, designing and training neural networks, and software development;

The winner is expected to demonstrate the ability to:

- understand graph theory, modular arithmetic, real and complex analysis, and mathematical logic;
- understand the fundamental laws of probability distributions;
- solve standard differential equations (both ordinary and partial) and analyze the stability of systems;
- construct functions of random variables and apply limit theorems.

## 2. List of degree programs covered by the subject area

### 2.1. List of doctoral degree programs

1.1.5 Mathematical Logic, Algebra, Number Theory and Discrete Mathematics
1.2.1 Artificial Intelligence and Machine Learning
1.2.2 Mathematical Modelling, Numerical Methods and Software Complexes
1.2.3 Theoretical Computer Science, Cybernetics
2.3.1 Systems Analysis, Management and Information Processing, Statistics

### 2.2. List of master's degree programs

01.04.01 Mathematics
01.04.02 Applied Mathematics and Informatics
01.04.03 Mechanics and Mathematical Modelling
01.04.04 Applied Mathematics
02.04.01 Mathematics and Computer Science

## 3. Content

### Field of science 1. Applied Mathematics

1. Function: Ways of defining a function, basic elementary functions, elementary functions, polar coordinate system.
2. Continuity: Main properties of continuous functions at a point and on an interval; points of discontinuity.
3. Random variable, expected value, variance.
4. Functions: reversibility, composition, image/preimage, number of mappings
5. Combinatorics: permutations, arrangements, combinations, binomial coefficients, inclusion-exclusion.
6. Sequences: recursions, progressions, patterns, asymptotics.
7. Fundamentals of programming (data types, variables, operations).
8. Eigenvalues and eigenvectors: definition, characteristic equation, diagonalisation, geometry, multiplicity and defectiveness.
9. Matrices and determinants: rank and linear independence.

**Field of science 2. Computer Science and Artificial Intelligence**

1. Limits: Limit of a numerical sequence and its properties; limit of a function (Cauchy and Heine definitions); infinitesimals and infinities, their properties; comparison of infinitesimals; main theorems about limits; first and second remarkable limits.
2. Derivatives: Derivative of a function at a point; geometric meaning; differentiability and its relation to continuity and existence of derivative; derivatives of basic elementary functions, composite functions, implicitly and parametrically defined functions; higher-order derivatives.
3. Data Structures (arrays, lists, stacks, queues).
4. Algorithms and their complexity analysis (sorting, searching, O-notation).
5. Object-Oriented Programming (classes, inheritance, polymorphism).
6. Working with files and data formats (JSON, CSV, XML).
7. Numerical methods (solving nonlinear equations, interpolation).
8. Matrices and determinants: types of matrices, operations, determinant, inverse matrix.

**Field of science 3. Mathematics**

1. Main theorems of calculus: Fermat, Rolle, Lagrange, Cauchy, L'Hospital's Rule.
2. Taylor's formula, expansion of functions.
3. Function analysis: monotonicity, extrema, convexity, inflection points.
4. Functions of several variables: partial derivatives, gradient, extrema, Lagrange multipliers.
5. Integrals: indefinite, definite, applications, improper integrals.
6. Numerical and functional series: convergence, power series, Taylor series.
7. Multiple integrals: double and triple integrals, coordinate systems.
8. Line and surface integrals, Green's theorem, Ostrogradsky–Gauss theorem (divergence theorem), Stokes' theorem.
9. Fourier series, function expansions.
10. Complex numbers, functions of a complex variable, analyticity.
11. Integrals and series in complex analysis: Cauchy's theorem, Cauchy integral formula, Taylor and Laurent series.
12. Singular points, residues, applications to integrals.

**Field of science 4. Cybernetics**

1. Vector spaces: axioms, linear dependence, basis, dimension, linear mappings, transformation matrices, kernel and image.
2. Orthogonality and OLS: orthonormal bases, matrices, Gram–Schmidt, least squares method, application in regression.
3. Systems of linear equations: forms of notation, methods of solution (Gauss, Jordan), Kronecker–Capelli theorem, structure of solutions.
4. Transfer function matrix: A, B, C, D models, connection with differential equations, stability through eigenvalues.
5. Sets: set algebra, Cartesian product, set power, applications in computer science and probability
6. Boolean algebra: laws, CNF and CNF.

**Field of science 5. Mathematical Logic**

1. Relations: equivalence, order, matrices and graphs, properties
2. Graphs: types, connectivity, trees, DFS/BFS, planarity, bipartiteness.
3. Venn diagrams: intersections, logical formulas, analysis of complex expressions.
4. Permutations: with repetitions, inversions, cycles, parity, optimisation.

**Field of science 6. Statistics and Probability Theory**

1. Random variables, their distribution functions (discrete and continuous), mathematical expectation and variance, properties and transformations.

2. Basic distributions of random variables: binomial, geometric, uniform (continuous), Poisson, exponential, normal.
3. Estimation of numerical characteristics of distributions: point estimates, sample mean and variance, confidence intervals.
4. Fundamentals of combinatorics: rules of addition and multiplication, permutations, arrangements, combinations with and without repetitions, binomial coefficients, Newton's binomial formula.

## 4. Preparation materials

### 4.1 Recommended reading

**Field of science 1. Applied Mathematics**

| **Reading list in English** |
|---|
| Bugrov Ya.S., Nikolsky S.M. Differential and Integral Calculus. Imported Pubn, 1983. 464 p. URL://https://catalog.princeton.edu/catalog/SCSB-8492201 |
| Christofides N. Graph Theory. An Algorithmic Approach. London, 1975. 415 p. URL://https://djvu.online/file/YisZ9w23dBmHW |
| Elsgolts L. Differential Equations and the Calculus of Variations. University Press of the Pacific, 2003. 450 p. URL://https://archive.org/details/ElsgoltsDifferentialEquationsAndTheCalculusOfVariations |
| Kleene S. C. Mathematical logic. NY: Dover Publ., 2002. 432 p. URL: https://archive.org/details/KleeneMathematicalLogic/Kleene-MathematicalLogic/ |

**Field of science 2. Computer Science and Artificial Intelligence**

| **Reading list in English** |
|---|
| Bugrov Ya.S., Nikolsky S.M. Differential and Integral Calculus. Imported Pubn, 1983. 464 p. URL://https://catalog.princeton.edu/catalog/SCSB-8492201 |
| Goodfellow I., Bengio Y., Courville A. Deep Learning. MIT Press, 2016. URL:// https://vk.com/wall-158423286_42375?ysclid=mcjdwcozub242831612 |
| Russell S., Norvig P. Artificial Intelligence: A Modern Approach, 4th ed. Pearson, 2022. URL:// https://vk.com/wall-206723877_11201?ysclid=mcjdxi719n326085567 |
| VanderPlas J. Python Data Science Handbook. O'Reilly, 2016. URL://https://github.com/jakevdp/PythonDataScienceHandbook |

**Field of science 3. Mathematics**

| **Reading list in English** |
|---|
| Bugrov Ya.S., Nikolsky S.M. Differential and Integral Calculus. Imported Pubn, 1983. 464 p. URL://https://catalog.princeton.edu/catalog/SCSB-8492201 |
| Ilyin V. A., Poznyak E. G. Analytic geometry. Moscow: Mir, 1984. 232 p. URL:// https://lib.ugent.be/en/catalog/rug01:000325357 |
| Shabunin M. I., Sidorov Yu. V., Fedoryuk M. V. Lectures on the Theory of Functions of a Complex Variable. Mir, 1985. URL://https://archive.org/details/SidorovFedoryukShabuninLecturesOnTheTheoryOfFunctionsOfAComplexVariable |
| Vinberg E.B. A course in algebra. American Mathematical Society, 2003. URL://http://www.ams.org/books/gsm/056/ |

**Field of science 4. Cybernetics**

| Reading list in English |
|---|
| Cybernsetics Unveiled: Exploring Its Interdisciplinary Nature and Implications URL:// https://www.prodigitalweb.com/cybernetics-interdisciplinary-nature impact/ |
| Dorf R.C., Bishop R.H. Modern Control Systems. Pearson Education, Inc., 2007. 1056 p. URL:// https://djvu.online/file/q8S2InQgW57hN |
| Kleene S. C. Mathematical logic. NY: Dover Publ., 2002. 432 p. URL: https://archive.org/details/KleeneMathematicalLogic/Kleene-MathematicalLogic/ |
| Novikov D.A. Cybernetics: From past to Future. Springer Cham, 2016. 107 p. URL:// https://www.researchgate.net/publication/287319297 |

**Field of science 5. Mathematical Logic**

| Reading list in English |
|---|
| Hurley, Patrick J. A concise introduction to logic. Cengage Learning, 2023. 658 p. URL: https://vk.com/wall-169240848_34036?ysclid=mcjeeae4s0776580937 |
| Kleene S. C. Mathematical logic. NY: Dover Publ., 2002. 432 p. URL: https://archive.org/details/KleeneMathematicalLogic/Kleene-MathematicalLogic/ |
| Mendelsohn E., Introduction to Mathematical Logic: New York: Chapman and Hall/CRC, 6th Edition, 2015, 513 p. URL: https://sistemas.fciencias.unam.mx/~lokylog/images/Notas/la_aldea_de_la_logica/Libros_notas_varios/L_02_MENDELSON,%20E%20-%20Introduction%20to%20Mathematical%20Logic,%206th%20Ed%20-%20CRC%20Press%20(2015).pdf |

**Field of science 6. Statistics and Probability Theory**

| Reading list in English |
|---|
| Feller W. An Introduction to Probability Theory and its Applications. John Wiley & Sons, 1967. URL:// https://www.amazon.com/Introduction-Probability Theory-Applications-Vol/dp/0471257087 |
| Ross, Sheldon M. Introduction to Probability and Statistics for Engineers and Scientists Academic Press 2014. 686 p. URL:// https://minerva.it.manchester.ac.uk/~saralees/statbook3.pdf |
| Walpol R.E., Myers R.H., Myers S.L. Probability and Statistics. Prentice Hall, 2011. URL:// https://archive.org/details/probabilitystati0000unse_l0c0_9thed |

## 4.2 Recommended online courses

**Field of science 1. Applied Mathematics**

| Online courses in English | Link | Course description |
|---|---|---|
| Linear algebra (Khan Academy) | https://www.khanacademy.org/math/linear-algebra | The course introduces the fundamental concepts and principles of linear algebra, including vectors, spans, linear transformations, matrices, determinants, eigenvectors, and related topics. |
| Calculus: Single Variable Part 1 – Functions. | https://www.coursera.org/learn/single-variable-calculus | The course covers the following topics: Taylor series, limits, |

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| | | L'Hospital's rule, and asymptotics. |
| Calculus: Single Variable Part 2 – Differentiation. | https://www.coursera.org/learn/differentiation-calculus | This course covers key topics such as derivatives, differentiation rules, linearization, higher-order derivatives, optimization, differentials, and differentiation operators. |
| Calculus: Single Variable Part 3 – Integration. | https://www.coursera.org/learn/integration-calculus | The course covers integrating differential equations, techniques of integration, the fundamental theorem of integral calculus, and difficult integrals. |
| Calculus: Single Variable Part 4 – Applications. | https://www.coursera.org/learn/applications-calculus | This introductory calculus course covers essential concepts including derivatives and integrals, with practical applications in calculating areas, volumes, and solving problems across geometry, physics, and other disciplines. Additionally, the course provides an introduction to probability theory, focusing on key ideas such as averages and mass. |

**Field of science 2. Computer Science and Artificial Intelligence**

| Online courses in English | Link | Course description |
|---|---|---|
| Calculus: Single Variable Part 1 – Functions. | https://www.coursera.org/learn/single-variable-calculus | The course covers the following topics: Taylor series, limits, L'Hopital's rule, and asymptotics |
| Calculus: Single Variable Part 2 – Differentiation. | https://www.coursera.org/learn/differentiation-calculus | This course covers key topics such as derivatives, differentiation rules, linearization, higher-order derivatives, optimization, differentials, and differentiation operators. |
| Calculus: Single Variable Part 3 – Integration. | https://www.coursera.org/learn/integration-calculus | The course covers integrating differential equations, techniques of integration, the fundamental theorem of integral calculus, and difficult integrals. |
| Calculus: Single Variable Part 4 – Applications. | https://www.coursera.org/learn/applications-calculus | This introductory calculus course covers essential concepts, including derivatives and integrals, with practical applications in calculating areas, |

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| | | volumes, and solving problems across geometry, physics, and other disciplines. Additionally, the course provides an introduction to probability theory, focusing on key ideas such as averages and mass. |
| Supervised Machine Learning: Regression and Classification | https://www.coursera.org/learn/machine-learning | Students will learn to develop machine learning models in Python using widely adopted libraries such as NumPy and scikit-learn. The course covers the creation and training of supervised learning models for forecasting and binary classification tasks, including linear regression and logistic regression. |

**Field of science 3. Mathematics**

| Online courses in English | Link | Course description |
|---|---|---|
| Calculus: Single Variable Part 1 – Functions. | https://www.coursera.org/learn/single-variable-calculus | The course covers the following topics: Taylor series, limits, L'Hopital's rule, and asymptotics |
| Calculus: Single Variable Part 2 – Differentiation. | https://www.coursera.org/learn/differentiation-calculus | This course covers key topics such as derivatives, differentiation rules, linearization, higher-order derivatives, optimization, differentials, and differentiation operators. |
| Calculus: Single Variable Part 3 – Integration. | https://www.coursera.org/learn/integration-calculus | The course covers integrating differential equations, techniques of integration, the fundamental theorem of integral calculus, and difficult integrals. |
| Calculus: Single Variable Part 4 – Applications. | https://www.coursera.org/learn/applications-calculus | This introductory calculus course covers essential concepts, including derivatives and integrals, with practical applications in calculating areas, volumes, and solving problems across geometry, physics, and other disciplines. Additionally, the course provides an introduction to probability theory, focusing on key ideas such as averages and mass. |
| Introduction to Complex Analysis (Coursera) | https://www.coursera.org/learn/complex-analysis | The course covers the following topics: complex numbers, power |

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| | | series, Laurent series and conformal mappings. |

**Field of science 4. Cybernetics**

| Online courses in English | Link | Course description |
|---|---|---|
| Introduction to Partial Differential Equations (MIT OpenCourseWare) | https://ocw.mit.edu/courses/18-152-introduction-to-partial-differential-equations-fall-2011/ | This course introduces three main types of partial differential equations: diffusion, elliptic, and hyperbolic. It includes mathematical tools, real-world examples and applications. |
| Linear Algebra from Elementary to Advanced | https://www.coursera.org/specializations/linear-algebra-elementary-to-advanced | This three-part course introduces linear algebra from the basics, requiring no prior knowledge or calculus. It covers linear systems, matrices, determinants, eigenvalues, symmetric matrices, and quadratic forms — with equal focus on algebraic and geometric perspectives.<br>Upon completion, students will be ready for advanced topics in data science, AI, machine learning, finance, math, computer science, and economics. |
| Discrete Mathematics | https://www.coursera.org/learn/discrete-mathematics | This course introduces fundamental mathematical objects—sets, functions, relations, and graphs—that are widely used in computer science. It emphasizes the development of skills in constructing and understanding rigorous proofs. While the course maintains a high level of rigor, it avoids excessive formalism: each concept is accompanied by a non-trivial result and its complete proof to illustrate practical application.. |

**Field of science 5. Mathematical Logic**

| **Online courses in English** | **Link** | **Course description** |
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| Introduction to Mathematical Logic (Coursera) | https://www.coursera.org/learn/mathematical-logic | The course explores advanced topics in mathematical logic and foundational studies, including formal systems, Gödel's incompleteness theorems, and the fundamentals of set theory. |
| Logic: The Basics by Stanford University | https://www.coursera.org/learn/logic | The course covers propositional and predicate logic, proof methods, and applications in computer science. |
| Introduction to Logic by University of Washington | https://www.coursera.org/learn/logic-uw | The course focuses on propositional and predicate logic, emphasizing the application of logical reasoning to problem-solving across various domains. |
| Reasoning Under Uncertainty by MIT | https://www.edx.org/course/reasoning-under-uncertainty-mitx-6-034-1x | This course explores the application of logic to problems involving reasoning under uncertainty, with a focus on its relevance to artificial intelligence. |

**Field of science 6. Statistics and Probability Theory**

| **Online courses in English** | **Link** | **Course description** |
|---|---|---|
| Introduction to Statistics (Coursera) | https://www.coursera.org/learn/stanford-statistics | This course provides a foundational introduction to statistics, with a focus on understanding and applying statistical reasoning in various contexts. Topics include descriptive statistics, sampling and randomized controlled experiments, probability, sampling distributions and the central limit theorem, regression analysis, common tests of significance, resampling techniques, and multiple comparisons. The course equips students with essential tools for analyzing data, interpreting results, and making data-driven decisions across diverse fields. |
| Probability Theory: Foundation for Data Science (Coursera) | https://www.coursera.org/learn/probability-theory-foundation-for-data-science | This course introduces the foundations of probability and its connection to statistics and data science. Students will learn how to calculate probabilities, distinguish |

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| | | between independent and dependent outcomes, and understand conditional events. The course covers both discrete and continuous random variables and their relevance to data collection. It concludes with an introduction to Gaussian (normal) random variables and the Central Limit Theorem, providing a basis for further study in statistical analysis. |
| Probability Theory | https://stepik.org/course/52134/promo | The course includes the basic concepts of probability theory, the most important probability-theoretic models, limit theorems and some methods of mathematical statistics. |
| Combinatorics and Probability | https://www.coursera.org/learn/combinatorics | This course explores a wide range of combinatorial problems, emphasizing the ability to identify and apply these concepts in real-world scenarios and algorithmic tasks. Students will gain hands-on experience with the recursive counting technique and develop a solid foundation in probability theory, fostering both theoretical knowledge and practical intuition. |
| To p or not to p? | https://www.coursera.org/learn/probability-statistics | This course equips students with essential tools for understanding uncertainty and making informed decisions. It introduces methods for quantifying uncertainty through probability and descriptive statistics, along with techniques for estimating averages and proportions. The fundamentals of hypothesis testing are also covered. Emphasis is placed on the application of core statistical concepts in multidimensional contexts, enabling students to make confident, data-driven decisions in a wide range of real-world scenarios. |