Engineering & Technology
Sample test

Task 1 (2 points)

A wheel rotates on an axis according to the law:

$\varphi(t) = 2t^2$(rad).

Find its angular velocity at t = 0.5 (s).

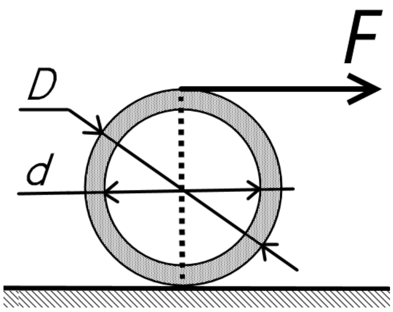
Solution:

$$\omega = \frac{d\varphi(t)}{dt} = 4t = \mathbf{2\ rad/s}$$

Answer: 2 rad/s

Task 2 (4 points)

A force F = 10 N was applied to the upper part of a pipe on a horizontal plane. The pipe has inner diameter d = 0.1 m, outer diameter D = 0.2 m and mass m = 40 kg. Calculate the angular acceleration of the pipe. There is no slipping between the pipe and the surface. Express the result in (rad/s^2), rounded to an integer.



Solution:

The theorem of angular momentum change holds:

$$\frac{dL}{dt} = M_E$$

where M_E is the external torque.

The torques at the point of contact between the pipe and the surface will be:

$$M_E = FD = I_0\varepsilon$$

where the moment of inertia of the pipe is determined by the expression:

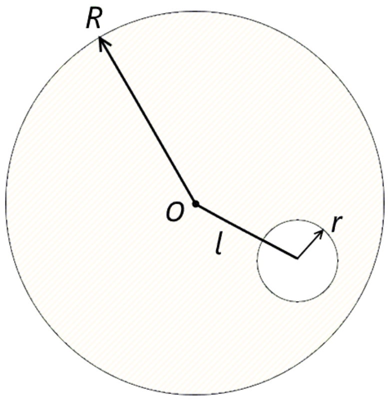
$$I_0 = \frac{1}{8}m(3D^2 + d^2)$$

Thus, the angular acceleration is:

$$\varepsilon = \frac{8FD}{m(3D^2 + d^2)} = 3(rad/s^2)$$

Task 3 (11 points)

1. A uniform disc of radius R = 0.6 m and mass M = 800 kg has a circular cavity of radius r = 0.2 m, located at distance l = R / 2 from the axis of rotation. Calculate the moment of inertia of the disk on axis O. Express the result in kg·m², rounded to an integer.



Solution:

Imagine that the cavity is filled with the material of the disc. The mass of this material is m. The mass of the "cavity" is to the mass of the disc without the cavity as the area of a circle of radius R is to the area of that of radius r. Hence:

$$\frac{m}{m+M} = \frac{\pi r^2}{\pi R^2} = m = \frac{Mr^2}{R^2 - r^2}$$

The moment of inertia of the disk with the cavity equals the moment of inertia of the disk of radius R and mass M + m without the mass of the "cavity":

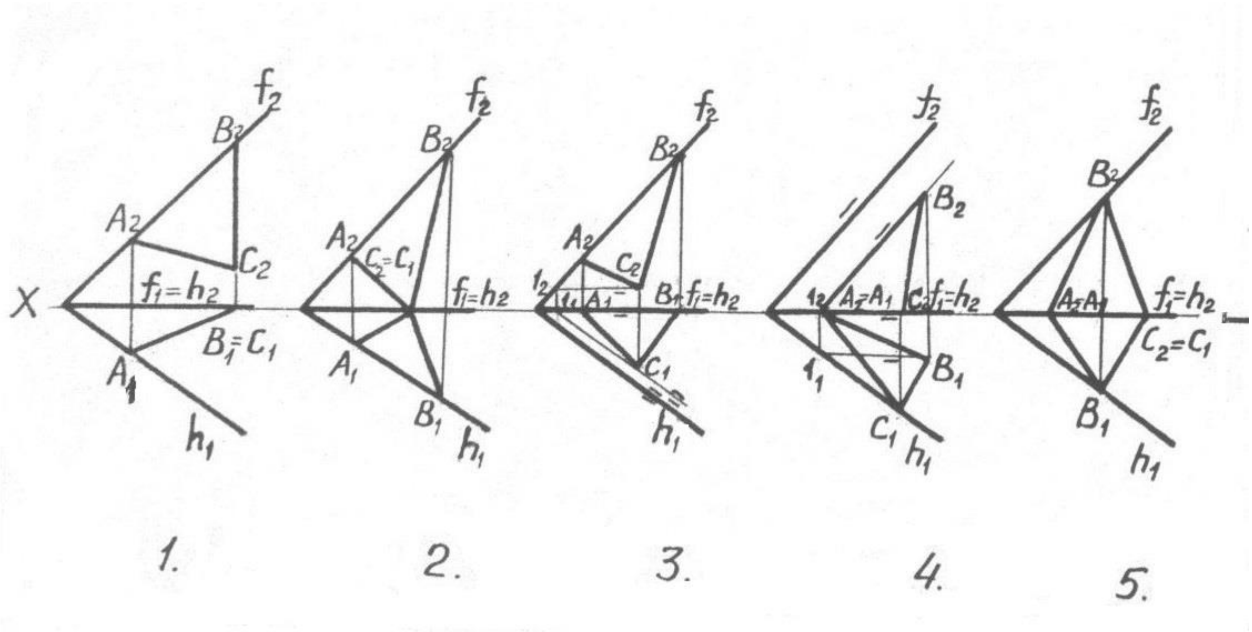
$$I = \frac{(M+m)R^2}{2} - \left(m\left(\frac{R}{2}\right)^2 + \frac{mr^2}{2}\right) = \frac{(M+m)R^2}{2} - \frac{Mr^2}{R^2 - r^2}\left(\frac{R^2 + 2r^2}{4}\right)$$

$$= 151(kg \cdot m^2)$$

$$I_0 = \frac{M}{R^2 - r^2}\left(\frac{R^4}{2} - \frac{r^4}{2} - r^2 l^2\right)$$

Task 4 (3 points)

In which figure is Δ ABC perpendicular to the horizontal plane of the projections?

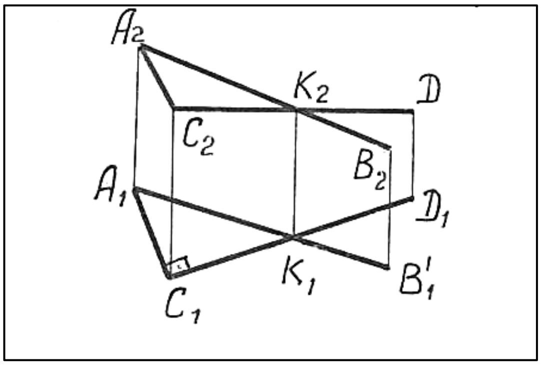


Answer: 1.

Task 5 (3 points)

What is line AC drawn in the plane defined by intersecting lines ∑ (AB, CD) called?

1. **horizontal**
2. frontal
3. profile line
4. the line of greatest slope

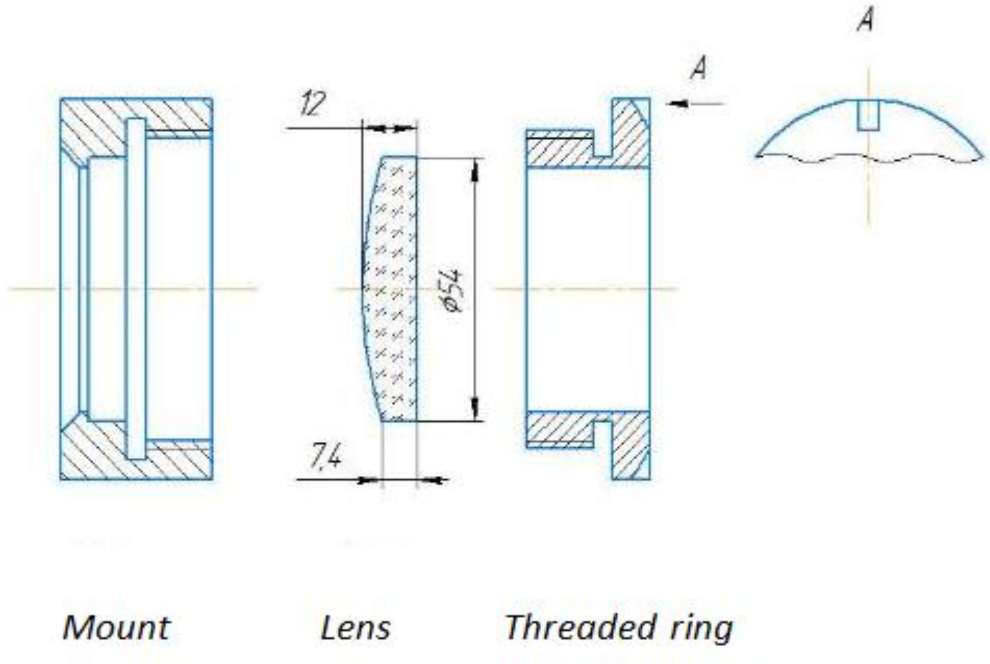


Task 6 (10 points)

Draw threaded connections

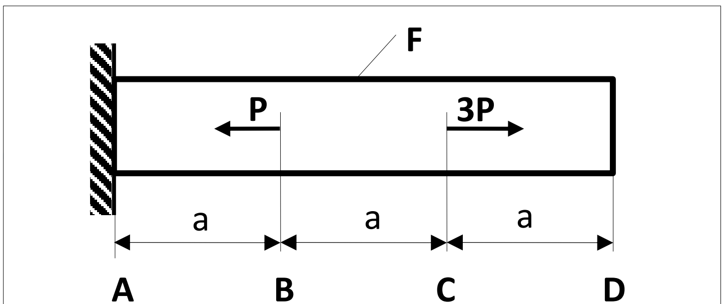
Draw the attachment of a lens to a mount with a threaded ring. Make an assembly drawing with a front section in place of the front view. Base the drawing on the 3D model of the assembly connection.

Solution:



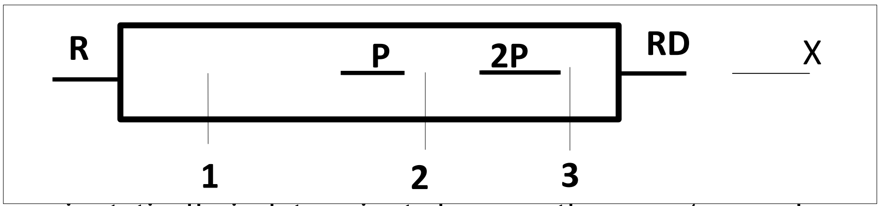
Task 7 (2 points)

A bar with a constant cross section of area F = 20 cm^2, whose left edge is rigidly fixed in section A, is loaded by concentrated forces P and $3P$ in sections B and C respectively (P = 10 kN). Determine the maximum modulo value of normal stresses σ_x^{max} in these sections.



Solution

1. Reactions R_A and R_D will occur in supports A and D. We can mark them on a force diagram.

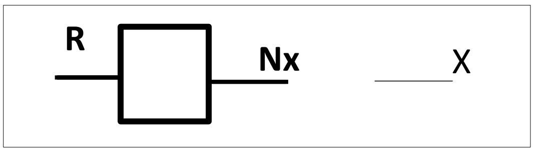


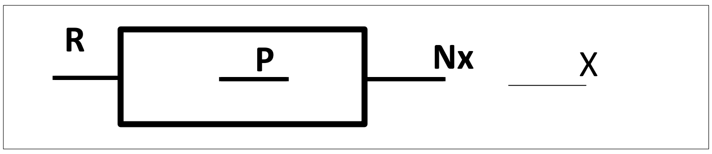
2. The beam is statically indeterminate because there are four unknown forces. If support D is dropped, reaction R_D will be considered as an external force. The additional equation $U_X(D) = 0$ will be:

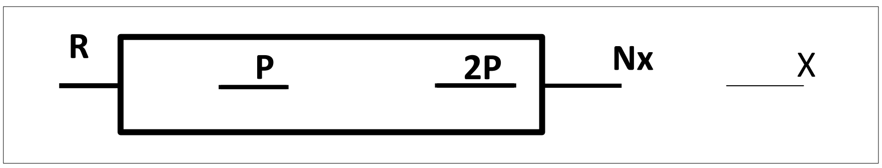
$$-\frac{P\cdot 2a}{F\cdot E}+\frac{2P\cdot 3a}{F\cdot E}-\frac{R_D\cdot 4a}{F\cdot E}=0 \Rightarrow R_D = P,\ R_A = 0.$$

3. To calculate axial forces Nx in the sections, we use the section method

Section 1:







$\Sigma X=0 \Rightarrow R_A - P + 2P + N_{x3} = 0, \; N_{x3} = -P = -10$ kN.

4. The maximum value (modulo) of axial force Nx in the force sections is 10 kN. Since the cross section of the bar is constant, the maximum modulo value of the normal stress is:

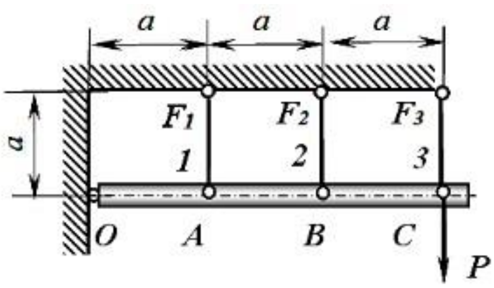
$\sigma^{max} = \frac{|N_x|}{F}$ = 10 kN /20cm^2=15 MPa

Answer: 15 MPa.

Task 8 (4 points)

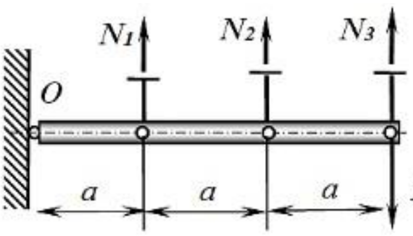
Beam ***OC*** is suspended on rods ***1, 2, 3*** made of a material with Young's modulus E. The rods have cross-sectional areas F_1, F_2 ***and*** F_3 respectively; the length of the rods is ***a***. The distance from the joint and between the rods is also ***a***. The beam is considered a rigid body compared to the rods. The left end of the beam is attached to a rigid wall at point ***O*** with a hinged connection. At point ***C,*** it is loaded with force ***P***. The weight of the beam is neglected compared to the force ***P***. ***P*** = 14 kN, $F1=F2=F3=F$.

Determine the value of axial force N_1 in rod ***1***. Give the answer in (kN), rounded to an integer.



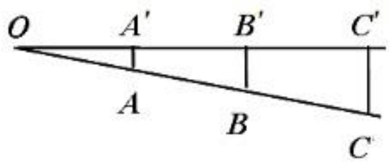
Solution:

Since the beam is in equilibrium under load, we can cut it out along the rods near the beam and obtain an equilibrium equation, assuming that all the rods experience tension.



$\Sigma M_o = 0 \Rightarrow N_1 \cdot a + N_2 \cdot 2a + N_3 \cdot 3a = P \cdot 3a$ (1)

Then we build a deformed system. Beam ***OC*** will rotate on hinge ***O*** under force P. The amount of rotation of the beam is determined by the deformations of rods ***1, 2*** and ***3***. Due to the insignificance of deformations, we can assume that any point of the beam moves vertically downward.



In the figure, ***OC′*** demonstrates the position of the beam without load and ***OC shows*** that under load ***P.*** Accordingly, ***AA′*** is the elongation of rod ***1; BB′***, of rod 2; ***CC′***, of ***3***. Now we can write down the geometric similarity of the elongation of the rods.

$$\Delta l_2 = 2\Delta l_1 \quad \text{и} \quad \Delta l_3 = 3\Delta l_1 \quad (2)$$

Then we express the elongation of rods through forces, lengths, and stiffness.

$$\Delta l_1 = N_1 \cdot a / EF$$

$$\Delta l_2 = N_2 \cdot a / EF \qquad (3)$$

$$\Delta l_3 = N_3 \cdot a / EF$$

By replacing expressions for extensions (3) in (2), we obtain:

$$N_2 = 2N_1 \quad \text{и} \quad N_3 = 3N_1 \qquad (4)$$

Next, we substitute expression (4) into (1)

$$N_1 = \frac{3}{14} \cdot P \qquad N_2 = \frac{6}{14} \cdot P \qquad N_3 = \frac{9}{14} \cdot P \quad (5)$$

By substituting the load value ***P*** = 14 kN, we get the amount of force in rod 1: N_1 = 3 kN.

Answer: 3

Task 9 (11 points)

A symmetric system of three rods ***1***, ***2*** and ***3*** made from a material with Young's modulus E has cross-sectional areas ***F1***, ***F2***, and ***F3*** respectively. The length of vertical rod

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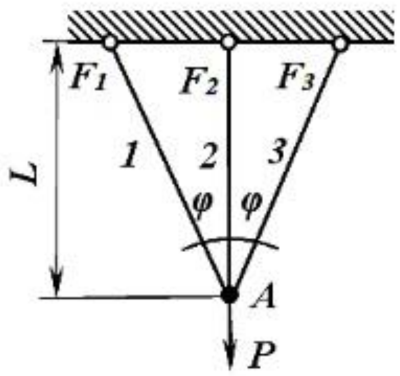
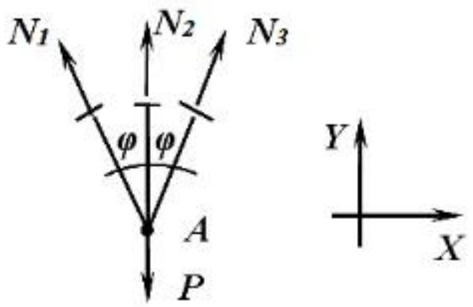
2 is ***L***. The angles between the rods are the same and equal to φ. The upper ends of the rods are attached to a rigid wall with a hinged connection, whilst the lower ends are similarly attached at point ***A***, to which vertical force ***P*** is applied. $P = 10$ kN, $F1 = F2 = F3 = F$, $\cos \varphi = 0.7$.

Calculate force ***N1*** in rod ***1***.



Solution:

Since the system of rods under load ***P*** is in equilibrium, we can cut the rods near node ***A*** and obtain equilibrium equations, assuming that all the rods experience tension.



$\Sigma X = 0 \Rightarrow -N_1 \cdot \sin\varphi + N_3 \cdot \sin\varphi = 0,$

and $N_1 = N_3$ since the system is symmetrical.

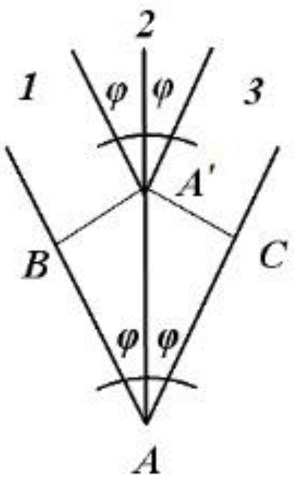
$\Sigma Y = 0 \Rightarrow -P + N_1 \cdot \cos\varphi + N_2 + N_3 \cdot \cos\varphi = 0,$

if $N_1 = N_3$, $-P + 2N_1 \cdot \cos\varphi + N_2 = 0$ (1)

Then we build a deformed system. Under force ***P***, the node moves from position ***A***' to ***A***. The amount of movement of the node is determined by the deformations of rods ***1***, ***2*** and ***3***. Due to the insignificance of the deformations, we can assume that angle φ between the rods does not change.

So, ***A'A*** will be the extension of rod 2; ***AB***, of rod 1; ***AC***, of rod ***3***. The relation between the sides of the right triangle ***A'AB*** will be:

$$\Delta l_1 = \Delta l_2 \cdot \cos\varphi \qquad (2)$$



Then, we express the elongation of rods **1** and **2** in terms of forces, lengths, and stiffness to obtain

$$\Delta l_1 = N_1 \cdot l / EF \cos\varphi \quad \text{и} \quad \Delta l_2 = N_2 \cdot l / EF \qquad (3)$$

By substituting expressions for extensions (3) into (2), we get

$$N_2 = N_1 / \cos^2\varphi \qquad (4)$$

Next, we substitute expression (4) into (1) and get

$$N_1 = P \cdot \cos^2\varphi / (1 + 2\cos^3\varphi) \quad \text{и} \quad N_2 = P / (1 + 2\cos^3\varphi) \qquad (5)$$

Substituting the value of load P = 10 kN, cos φ = 0.7, we obtain the amount of force in rod ***1***.

N_1 = 10 ·0.49 / 1.686 = 2.90 kN

Answer: 2.90 kN

Task 10 (2 points)
Mechanical properties of materials (evaluating the hardness of materials)

Find the Vickers hardness number if an applied load of 2.5 kgf leaves an indentation with an average diagonal of 0.389 mm.

Solution

The Vickers hardness test focuses on the relationship between the depth of penetration of a tetrahedral diamond pyramid (indenter) with a face angle of 136° into the material and the amount of applied force. The surface hardness of the test material can be determined using the formula:

$$HV = 1.8544\ P/d^2,\ kgf/mm^2 \text{ or hardness units,}$$

where P is the load on the indenter, kgf,
d is the average value of indentation diagonals, mm.

We substitute the given values of load applied to the indenter and the average value of the indentation diagonals into formula (1):

$$HV = 1.8544 \cdot 2.5\ kgf/(0.389\ mm)2 = 30{,}64\ kgf/mm2 \text{ or } 30{,}64HV$$

Task 11 (4 points)
Heat treatment of materials

Calculate the carbon content in steel needed to obtain a material with martensite hardness $HRC_M = 33$. What percentage of the area will be occupied by ferrite in the microstructure analysis of the initial material?

Solution:

During quenching, the hardness of steel increases as a martensitic structure is formed. Although the hardness of martensite depends on many parameters, the carbon content in steel is the most influential factor. To determine the carbon content in an alloy by the hardness of the resulting martensite after quenching, we can use the following empirical formula. (It is applied when calculating the carbon content in steels with martensite hardness in the range of 30-65HRC_M.)

$$C = 13.9 - 1.35855156 \cdot HRC_M + 0.0487592 \cdot HRC_M^2 - 0.0007566 \cdot HRC_M^3 + 0.00000432 \cdot HRC_M^4, (2)$$

where C is the carbon content, %,
HRC_M is the required hardness of martensite in hardness units.

To determine carbon content in steel at a given hardness of martensite, we use formula (2):

$$C = 13.9 - 1.35855156 \cdot 33 + 0.0487592 \cdot 332 - 0.0007566 \cdot 333 + 0.00000432 \cdot 334 = 0.1\%$$

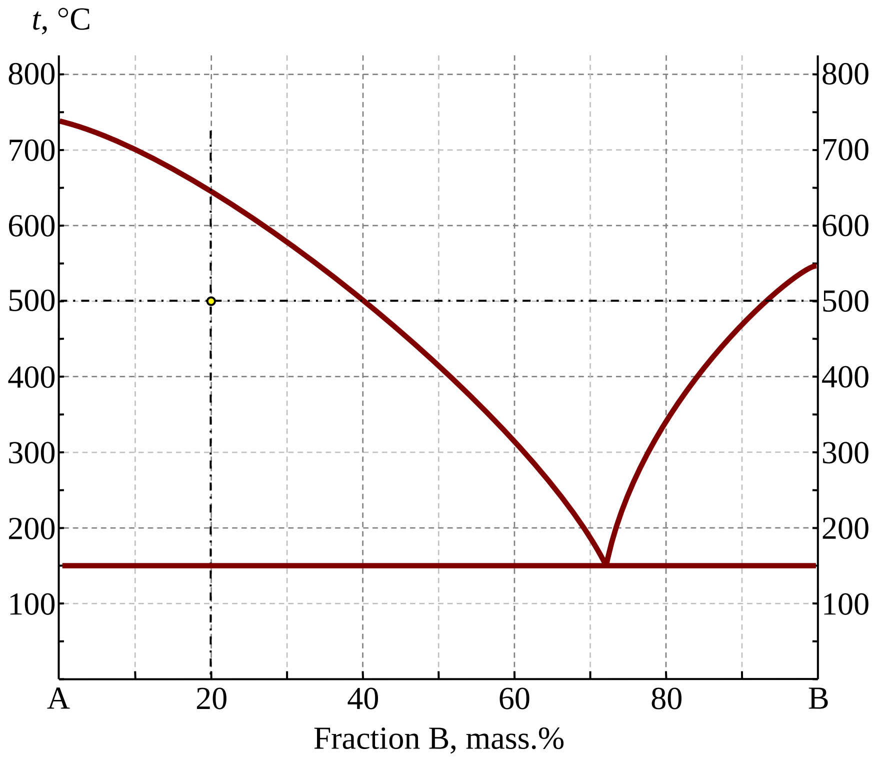
The steel contains 0.1 percent of carbon, i.e. it is a hypoeutectoid steel whose microstructure consists of ferrite and pearlite. To find the area occupied by perlite, we apply the following formula:

$$P = C \cdot 100/0.8 = 0.1 \cdot 100/0.8 = 12.5\%.$$

Consequently, the proportion of ferrite is F = 100 – P = 87.5%.

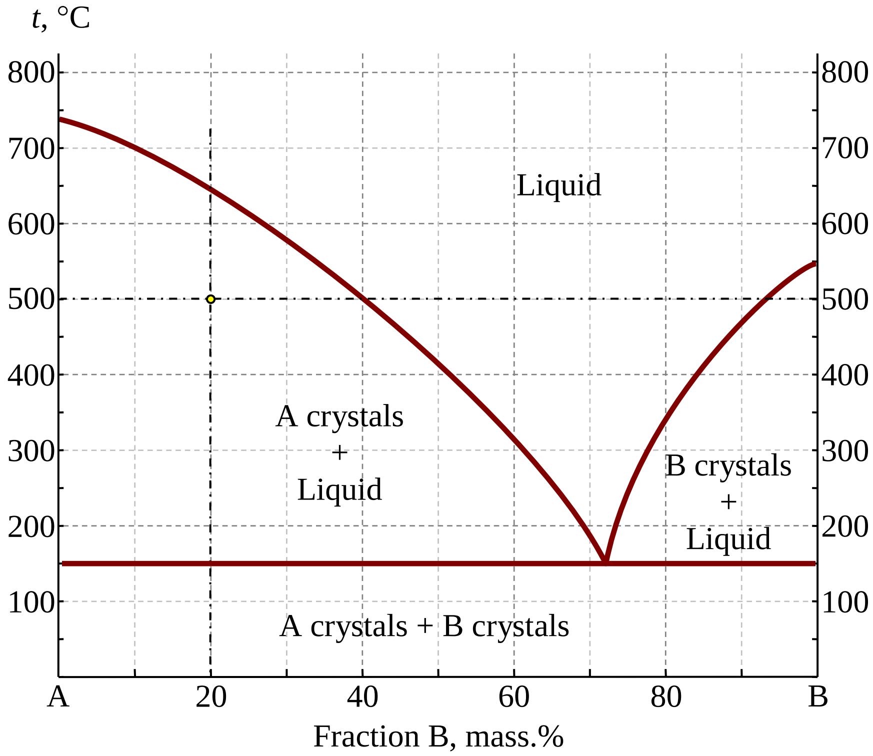
Task 12 (11 points)
Reading phase diagrams of a two-component system

1. Mark all fields on the phase diagram.
2. Draw the liquidus and solidus lines and mark characteristic points on the phase diagram.
3. Draw a cooling curve for the composition.
4. Specify the composition and mass ratio of the coexisting phases for the selected point.

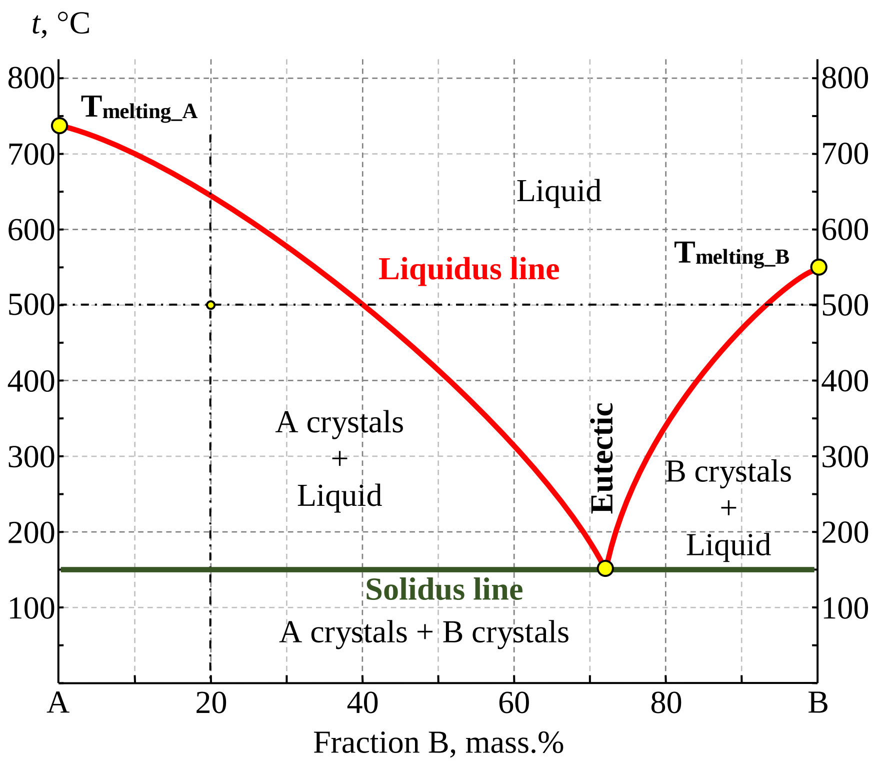


Solution. The figure shows a phase diagram of a simple eutectic type without mutual solubility of the components in the solid state. This system lacks A and B component compounds.

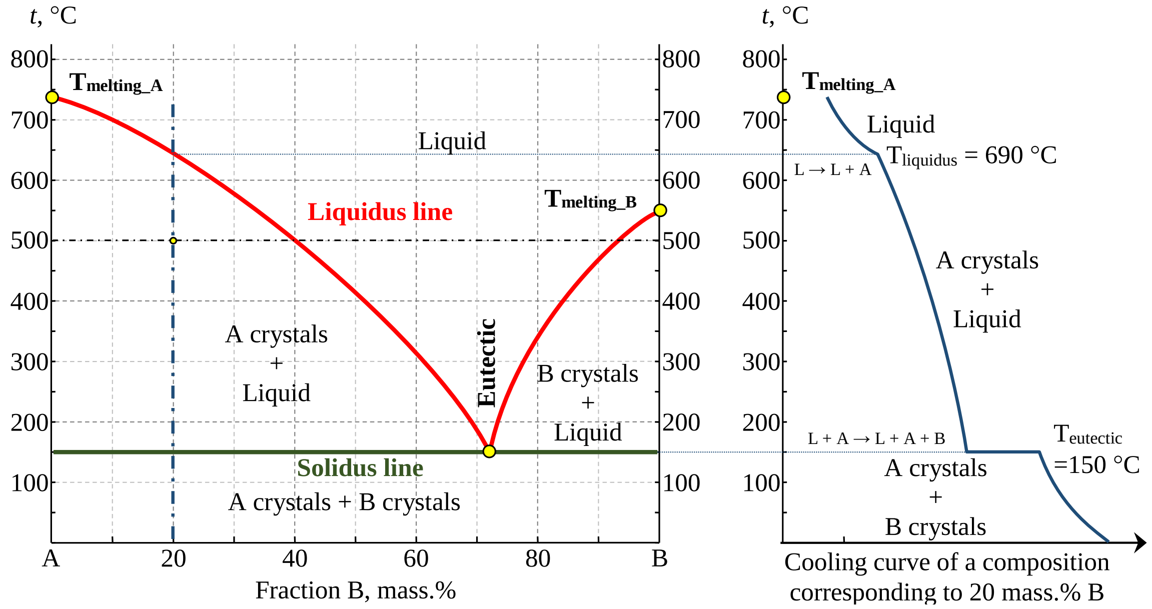
1. First, we label fields in the phase diagram:



2. Then, we draw the liquidus and solidus lines and mark the characteristic points:

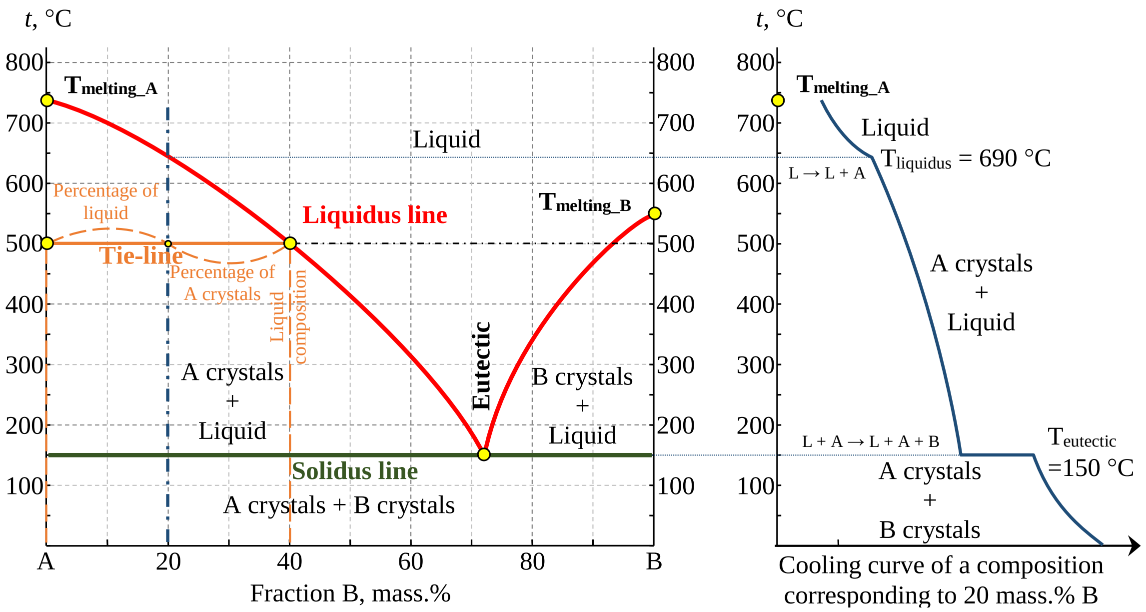


3. The cooling curve for the composition will look as follows:



4. For the imaging point, we identify the composition and mass ratio of coexisting phases. The imaging point lies in a two-phase field in which crystals of component

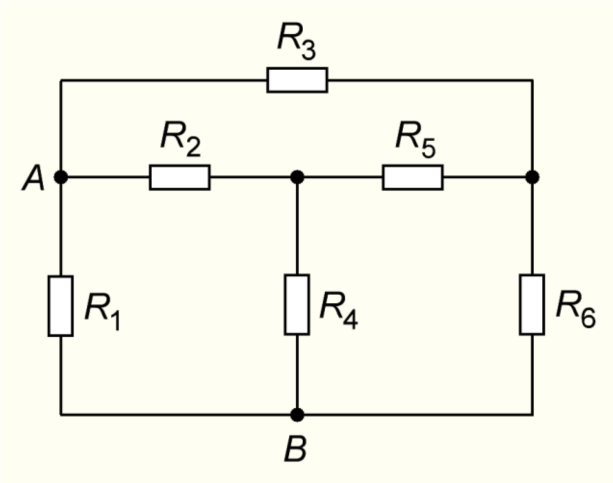
A are in equilibrium with the liquid phase. This point corresponds to a composition of 80 mass.% of component A and 20 mass.% of component B. At the imaging point, the system has a temperature of 500°C. To determine the composition and ratio of the coexisting phases, we draw a tie-line through the imaging point.



The tie-line suggests that the liquid at the imaging point corresponds to the composition of 60 mass.% of component A and 40 mass.% of component B. The ratio between the liquid and crystals A, determined by the lever rule, is 1:1 by mass.

Task 13 (2 points)

Look at the circuit diagram below. Determine equivalent resistance between nodes (points) A and B if $R_1 = 1\ \Omega$, $R_2 = R_3 = R_5 = 6\ \Omega$, $R_4 = R_6 = 2\ \Omega$:



A) 1 Ω

B) 0.8 Ω

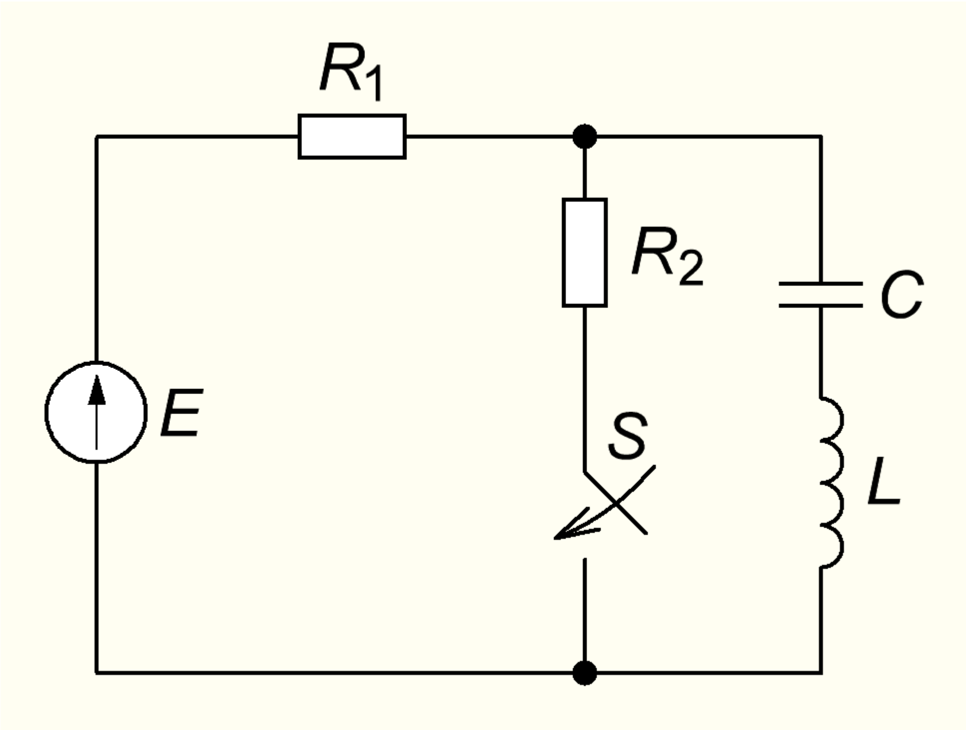
C) 1.25 Ω

D) 0

E) 23 Ω

Task 14 (4 points)

Look at the circuit diagram below. The source voltage is $E = const$. Choose one possible form of the obtained algebraic equation of the circuit:



A) $s^2 + 1 = 0$

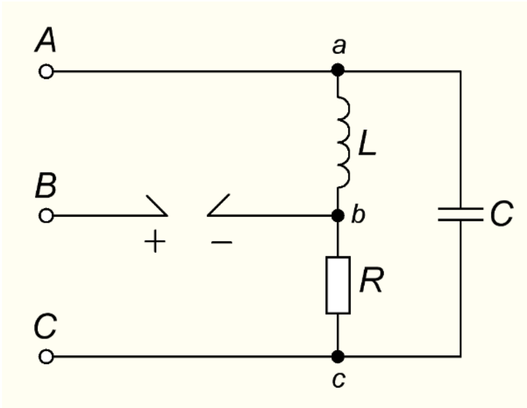
B)$s^2 - s - 1 = 0$

C)$s^2 + s + 1 = 0$

D) $s^3 + s^2 + s + 1 = 0$

E)$s^2 + s - 1 = 0$

Task 15 (10 points)
Look at the circuit diagram below. The three-phase source is symmetrical, and the phase sequence is clockwise. The line voltage is $100V$. Line wire breakage has occurred at phase B. Determine the voltage $u_{Bb}(t)$ if R= 10 Ω, Z_C ∨ 20Ω, Z_L ∨ 5Ω.



Solution. Since the voltage system is symmetric, we assume that the initial phase α_{AB}=120° and determine the complex line voltage in the circuit:

$$\grave{U}_{AB} = U_{Л} \cdot e^{j\cdot 120°} = 100 \cdot e^{j\cdot 120°},$$

$$\grave{U}_{BC} = U_{Л} \cdot e^{j\cdot 0°} = 100,$$

$$\grave{U}_{CA} = U_{Л} \cdot e^{-j\cdot 120°} = 100 \cdot e^{-j\cdot 120°}.$$

Then we find the complex resistances of the phases:

$$Z_{ab} = Z_L = j \cdot \omega \cdot L = j \cdot |Z_L| = 5j,$$

$$Z_{bc} = Z_R = R = 10,$$

$$Z_{ca} = Z_C = -j \cdot \frac{1}{\omega \cdot C} = -j \cdot |Z_C| = -20j.$$

The line wire has been broken at phase B. Thus, $\grave{I}_{ab} = \grave{I}_{bc}$. We find the former complex current using Kirchhoff's and Ohm's laws in complex form:

$$\grave{I}_{ab} = \frac{\grave{U}_{AC}}{Z_{ab} + Z_{bc}},$$

$$\grave{U}_{AC} = -\grave{U}_{CA} = 100 \cdot e^{j\cdot 60°},$$

$$\grave{I}_{ab} = \frac{100 \cdot e^{j\cdot 60°}}{10 + 5j} = \frac{100 \cdot e^{j\cdot 60°}}{5\sqrt{5} \cdot e^{j\cdot 26.565°}} = 4\sqrt{5} \cdot e^{j\cdot 33.435°}.$$

The complex phase voltage bc is found according to Ohm's law in complex form:

$$\grave{U}_{bc} = \grave{I}_{bc} \cdot Z_{bc} = 4\sqrt{5} \cdot e^{j\cdot 33.435°} \cdot 10 = 40\sqrt{5} \cdot e^{j\cdot 33.435°}.$$

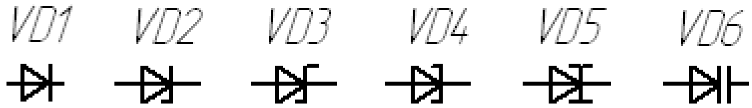
Using Kirchhoff' voltage law in complex form, we obtain the complex voltage $\grave{U}_{Bb}$

$$\grave{U}_{Bb} = \grave{U}_{BC} - \grave{U}_{bc} = 100 - 40\sqrt{5} \cdot e^{j \cdot 33.435°} \approx 25.359 - 49.282j = 55.424 \cdot e^{-j \cdot 62.771°}$$

Thus, $u_{Bb}(t) = 78.381 \cdot cos(\omega \cdot t - 62.771°)$

Answer: $u_{Bb}(t) = 78.381 \cdot cos(\omega \cdot t - 62.771°), V$

Task 16
(1 point)



Which of the symbols represents a Zener diode?

1. VD1
2. **VD2**
3. VD3
4. VD4
5. VD5
6. VD6

Task 17
(4 points)

The value of direct current flowing through a rectifier diode increased from 3mA to 16mA when the direct voltage changed from 0.2 V to 0.4 V.

Calculate the differential resistance of this rectifier diode.

1.15,38$k\Omega$

2.15,38Ω

3.25,00Ω

4.25,00MΩ

Solution: $R_{diff} = \frac{\Delta Udirect}{\Delta Idirect} = \frac{0,4-0,2}{(16-3)*10^{-3}} = 15{,}38\Omega$.

Task 18
(12 points)

Look at the circuit of a non-inverting amplifier (Fig.1). Find the voltage V_{out}, if V_{in}=60 mV, R_F=1000 Ω and R_G =200 Ω.

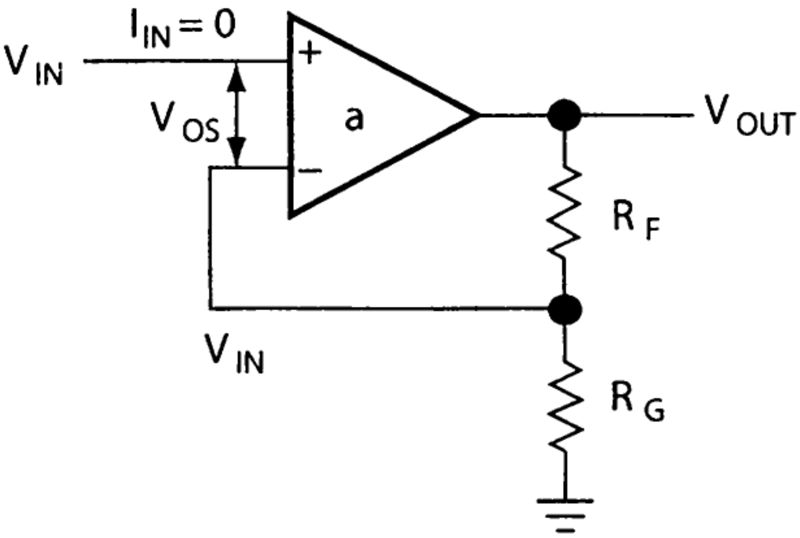


Fig.1

Solution: $Vout = Vin\left(1+\frac{RF}{RG}\right) = 60*10^{-3}*\left(1+\frac{1000}{200}\right) = 60*10^{-3}*6 = 360mV$