Program: Mathematics and Artificial Intelligence

This document outlines the scope of themes, which may be included in the Olympiad tests. The themes are grouped by areas and are followed by a list of recommended literature in the Russian and English languages.

Olympiad winner's skill set

Need to know:

1. Data analysis methods based on the principles of fundamental mathematics;
2. The fundamentals of how artificial intelligence works;
3. The fundamental concepts of modern mathematics.

Need to be able to:

1. Choose and justify methods for solving assigned tasks;
2. Use modern software for data processing;
3. Apply knowledge from across various branches of mathematics to obtain known solutions to research problems.

Need to possess the skills in:

1. Creating statistical reports, graphs, charts and diagrams, basic models for selected data;
2. Predicting the results of research activities (basic skills in formulating scientific hypotheses);
3. Proving well-known modern mathematical theorems (within the specialization).

Content

Section 1. Linear algebra and analytic geometry

1. Systems of linear algebraic equations. Cramer's rule. Rouche–Capelli theorem. General solution of a system of linear equations
2. Vector spaces. Basis, dimension of a vector space. Subspaces. Sum and intersection of vector subspaces. Linear transformation of a finite-dimensional vector space, the matrix of a linear transformation. Changing the matrix of a linear transformation with the change of basis
3. Eigenvectors and eigenvalues, their properties
4. Euclidean spaces. Linear transformations on Euclidean spaces
5. Bilinear and quadratic forms
6. Vectors and their properties. Dot (scalar), cross (vector) and triple scalar (mixed) products
7. Ways to define a line and a plane. Calculating the angle between a plane and a line, and between two lines. Calculating the distance from a point to a line and the distance between two lines
8. Second-order curves (conic sections). Ellipse, parabola, hyperbola
9. Second-order surfaces (quadric surfaces). Ellipsoid, one-sheeted hyperboloid, two-sheeted hyperboloid, elliptic paraboloid, hyperbolic paraboloid, cone
10. Affine transformations: definition and properties
11. Orthogonal transformations: definition and properties

Section 2. Real and complex analysis (calculus)

1. Limit of a sequence: definition and properties. Cauchy's criterion. Limit inferior and limit superior. The Bolzano–Weierstrass theorem. The limit of a function at a point. The equivalence of Cauchy's and Heine's definitions
2. Function continuity at a point. Behavior of a continuous function on a segment: the Weierstrass and Bolzano-Cauchy theorems. The inverse function theorem. Uniform continuity, Cantor's theorem.
3. Derivative of a function (of a single variable) at a point: definition and basic properties. The derivative of a composition of functions. Differentiability of a function at a point. Function differential at a point. Derivative of an inverse function. Higher order derivatives and differentials. The Leibniz rule. Rolle's Theorem, the mean value theorem, Cauchy's mean value theorem. L'Hôpital's rule. Taylor's expansion with the Peano and Lagrange form of the remainder. Using Taylor's expansion and L'Hôpital's rule for limit calculations. Using derivative to study the properties of a function of a single variable: monotonicity, extrema, convexity, inflection points
4. Indefinite integral. Definite integral. Darboux's criterion of the integrability of a function. Properties of an integral with a variable upper limit: continuity, differentiability. The Newton–Leibniz formula. Geometric applications of definite integral. Improper integrals. Absolute convergence and conditional convergence. Cauchy's criterion, Dirichlet's test of convergence
5. Differentiability of a function of several variables. Necessary conditions and sufficient conditions for differentiability. Gradient of a function. Implicit function theorem. Local extrema of a function of several variables. Necessary conditions and sufficient conditions of a local extremum point. Conditional extrema of a function. The Langrange multiplier method, necessary and sufficient conditions for conditional extrema
6. Numeric series. Absolute and conditional convergence. Cauchy's criterion, comparison test, integral test, the Root and Ratio tests, Leibniz's and Dirichlet's tests. Function series. Uniform convergence. Cauchy's criterion, the Weierstrass test, Dirichlet's test of uniform convergence. Power series. The radius of convergence, the Cauchy—Hadamard formula. The Taylor expansion. Taylor expansions for elementary functions
7. Line integral. Green's theorem. Surface integrals. The Gauss—Ostrogradsky theorem. Stokes' theorem
8. A Fourier series. Pointwise convergence conditions. Uniform convergence conditions. The Fourier transform of an absolutely integrable function and its properties. The Fourier transform of a derivative and the derivative of a Fourier transform
9. Complex numbers: definition, properties
10. Analytic functions
11. Integral of a function of a complex variable. Cauchy's integral theorem. Cauchy's integral formula
12. Maximum modulus principle and the Schwarz lemma
13. The Taylor and Laurent series
14. Residues of complex functions. Cauchy's residue theorem. Calculation of integrals using residues. Jordan's lemma
15. Conformal mappings. Möbius transformations. The Joukowsky transform

Section 3. Differential equations

1. Ordinary differential equations. Separation of variables. Reduction of order of the differential equation
2. Linear ODEs and systems of ODEs with constant coefficients
3. Linear ODEs and systems of ODEs with variable coefficients. A fundamental solution set. The Wrońskian, Liouville's formula. The variation of constants method
4. Equilibrium of an autonomous system of differential equations. Classifying the equilibria of linear autonomous second-order systems. Stability and asymptotic stability of equilibrium. First integrals of autonomous system of differential equations. Theorem on the number of independent first integrals
5. Linear PDEs. General solution and the Cauchy problem
6. Calculus of variations. The Euler–Lagrange equation. Necessary condition for a weak local extremum

Section 4. Theory of probability and mathematical statistics

1. Fundamentals of combinatorics: counting rules, addition and multiplication rules, combinations with and without repetition, binomial coefficients and Newton's binomial theorem
2. Probability space. Independent events. Summation theorem. Conditional probability. Collectively exhaustive events. Law of total probability. Bayes' formula
3. A random variable and its cumulative distribution function. Expectation and variance of a random variable: definition and properties
4. Basic classes of probability distributions: binomial, geometric, uniform, Poisson, exponential, normal. Bernoulli trials. Chebyshev's inequality. Law of large numbers
5. Joint probability distributions. Independence. Covariance. Correlation coefficient
6. Estimating the parameters of a distribution. Statistical hypothesis testing

Section 5. Machine learning

1. Regression analysis. Binary data classification problem. Linear regression model. Multiple linear regression. Ordinary least squares method. Calculating linear regression coefficients. L1 and L2 regularization
2. Metric space. Euclidean space. Taxicab geometry. The k-nearest neighbors algorithm. The weighted k-nearest neighbors algorithm
3. Bayes' theorem. Bayesian decision rule. Naive Bayes classifier
4. Decision tree. Decision rules. The Gini impurity. Information gain. Classification and regression trees. Ensemble learning for classification and regression. Bagging. Boosting
5. Regression scoring. Classification scores and margins. Performance of a binary classifier. The confusion matrix. Confusion-matrix-based performance measures. Analysis of the coefficient of determination and the adjusted coefficient of determination. The ROC-curve: calculation, properties, and interpretation. The ROC-curve of a perfect and random classifiers. ROC AUC. The ROC AUC calculation method
6. The mathematical model of an artificial neuron. Activation functions. Feed-forward neural networks. The mathematical model of a multi-layer neural network. Neural network training as an optimization problem. Backpropagation. Calculating the gradient of the objective function. Loss function. Types of loss functions. Comparison of the learning processes with different loss functions. Neural networks training algorithms. Vanilla gradient descent and stochastic gradient descent

7. Deep learning. Convolutional neural networks. Structure of the convolutional neural network. The purpose of the convolutional block
8. Unsupervised learning. Cluster analysis. Comparison of clustering approaches. Clustering problem statement. Clustering score. Methods for calculating the distance between clusters
9. Clustering with graphs. Clustering with minimum spanning forest. Prim's algorithm. Kruskal's algorithm
10. The k-means method. Problem statement and computational steps (with an example illustration). k-means initialization. The k-means++ algorithm. Choosing the number of clusters. The elbow method
11. Hierarchical agglomerative clustering. Dendrogram. Types of metrics in a feature space. Methods for calculating the distance between clusters. Ward's method
12. Dimensionality reduction. Principal component analysis. Independent component analysis

Section 6. Discrete Mathematics

1. Division with a remainder. GCD and LCM. The Euclidean algorithm. Diophantine equations. Prime numbers. Factorization methods
2. Positional notation of natural numbers. Algorithms for converting numbers between bases
3. Arithmetic of remainders. Linear comparisons. Chinese remainder theorem. System of residual classes
4. Euler's totient function. Fermat's little theorem. Euler's theorem. RSA encryption.
5. General formulas of combinatorics. Enumerative combinatorics. The inclusion-exclusion principle
6. Graph. Graph properties. Depth-first search and breadth-first search. Connectivity of graphs. Kosaraju's algorithm. Planar graph. Euler's theorem
7. An Eulerian trail in a graph. A Hamiltonian path in a graph. Dijkstra's algorithm. The Floyd-Warshall algorithm
8. Spanning trees in a graph. Prim's algorithm. Kruskal's algorithm
9. Boolean functions. Normal forms of Boolean functions. Duality of functions. Zhegalkin polynomials. Closed classes. Post's theorem
10. Logic of statements. Predicate logic. Resolution method
11. Formal languages and grammars. Context-free grammars. Automatic grammars. Finite automata. Determinacy. Kleene's theorem
12. Turing machine. Markov algorithms. Partially recursive functions

Recommended literature

Section 1. Linear algebra and analytic geometry

Sources in Russian	Corresponding topic
1. В.А. Ильин, Э.Г. Позняк. Аналитическая геометрия, любое издание, напр., М.: Физматлит, 2004. URL:https://www.labirint.ru/books/585885/ (not free)	Vectors and their properties. Dot (scalar), cross (vector) and triple scalar (mixed) products Ways to define a line and a plane. Calculating the angle between a plane and a line, and between two lines. Calculating the distance

	from a point to a line and the distance between two lines Second-order curves (conic sections). Ellipse, parabola, hyperbola Second-order surfaces (quadric surfaces). Ellipsoid, one-sheeted hyperboloid, two-sheeted hyperboloid, elliptic paraboloid, hyperbolic paraboloid, cone Affine transformations: definition and properties Orthogonal transformations: definition and properties
2. В.А. Ильин, Э.Г. Позняк. Линейная алгебра, любое издание, напр., М.: ФИЗМАТЛИТ, 2007. URL:https://obuchalka.org/20170924 96602/lineinaya-algebra-ilin-v-a-poznyak-e-g-2005.html (free)	Systems of linear algebraic equations. Cramer's rule. Rouche–Capelli theorem. General solution of a system of linear equations Vector spaces. Basis, dimension of a vector space. Subspaces. Sum and intersection of vector subspaces. Linear transformation of a finite-dimensional vector space, the matrix of a linear transformation. Changing the matrix of a linear transformation with the change of basis Eigenvectors and eigenvalues, their properties Euclidean spaces. Linear transformations on Euclidean spaces Bilinear and quadratic forms
3. Д. В. Беклемишев. Курс аналитической геометрии и линейной алгебры, любое издание, напр.,. М.: Физматлит, 2005. URL: http://mathdep.ifmo.ru/wp-content/uploads/2020/09/%D0%9A%D1%83%D1%80%D1%81-%D0%B0%D0%BD%D0%B0%D0%BB%D0%B8%D1%82%D0%B8%D1%87%D0%B5%D1%81%D0%BA%D0%BE%D0%B9-%D0%B3%D0%B5%D0%BE%D0%BC%D0%B5%D1%82%D1%80%D0%B8%D0%B8-%D0%B8-%D0%BB%D0%B8%D0%BD%D0%B5%D0%B9%D0%BD%D0%BE%D0%B9-%D0%B0%D0%BB%D0%B3%D0%B5%D0%B1%D1%80%D1%8B.-%D0%91%D0%B5%D0%BA%D0%BB%D0%B5%D0%BC%D0%B8%D	Systems of linear algebraic equations. Cramer's rule. Rouche–Capelli theorem. General solution of a system of linear equations Vector spaces. Basis, dimension of a vector space. Subspaces. Sum and intersection of vector subspaces. Linear transformation of a finite-dimensional vector space, the matrix of a linear transformation. Changing the matrix of a linear transformation with the change of basis Eigenvectors and eigenvalues, their properties Euclidean spaces. Linear transformations on Euclidean spaces Bilinear and quadratic forms Vectors and their properties. Dot (scalar), cross (vector) and triple scalar (mixed) products Ways to define a line and a plane. Calculating the angle between a plane and a line, and between two lines. Calculating the distance from a point to a line and the distance between two lines

1%88%D0%B5%D0%B2-%D0%94.%D0%92..pdf (free)	Second-order curves (conic sections). Ellipse, parabola, hyperbola Second-order surfaces (quadric surfaces). Ellipsoid, one-sheeted hyperboloid, two-sheeted hyperboloid, elliptic paraboloid, hyperbolic paraboloid, cone Affine transformations: definition and properties Orthogonal transformations: definition and properties
4. И.И. Привалов. Аналитическая геометрия. СПб.: Лань, 2010. URL: https://www.litres.ru/ivan-privalov/analiticheskaya-geometriya-40-e-izd-uchebnik-dlya-v-62697081/ (not free)	Vectors and their properties. Dot (scalar), cross (vector) and triple scalar (mixed) products Ways to define a line and a plane. Calculating the angle between a plane and a line, and between two lines. Calculating the distance from a point to a line and the distance between two lines Second-order curves (conic sections). Ellipse, parabola, hyperbola Second-order surfaces (quadric surfaces). Ellipsoid, one-sheeted hyperboloid, two-sheeted hyperboloid, elliptic paraboloid, hyperbolic paraboloid, cone Affine transformations: definition and properties Orthogonal transformations: definition and properties
5. И.М. Гельфанд. Лекции по линейной алгебре, любое издание, напр., М.: Добросвет, МЦНМО, 1998. URL:http://www.tka4.org/materials/lib/Articles-Books/General/LinearAlgebra&Geometry/gelfand.pdf (free)	Systems of linear algebraic equations. Cramer's rule. Rouche–Capelli theorem. General solution of a system of linear equations Vector spaces. Basis, dimension of a vector space. Subspaces. Sum and intersection of vector subspaces. Linear transformation of a finite-dimensional vector space, the matrix of a linear transformation. Changing the matrix of a linear transformation with the change of basis Eigenvectors and eigenvalues, their properties Euclidean spaces. Linear transformations on Euclidean spaces Bilinear and quadratic forms
6. П.С. Александров. Лекции по аналитической геометрии, любое издание, напр., СПб.: Лань, 2016. URL:https://www.ozon.ru/product/lektsii-po-analiticheskoy-geometrii-popolnennye-neobhodimymi-	Vectors and their properties. Dot (scalar), cross (vector) and triple scalar (mixed) products Ways to define a line and a plane. Calculating the angle between a plane and a line, and between two lines. Calculating the distance

svedeniyami-iz-algebry-aleksandrov-320039875/?sh=dK3WCN9s_g (not free)	from a point to a line and the distance between two lines Second-order curves (conic sections). Ellipse, parabola, hyperbola Second-order surfaces (quadric surfaces). Ellipsoid, one-sheeted hyperboloid, two-sheeted hyperboloid, elliptic paraboloid, hyperbolic paraboloid, cone Affine transformations: definition and properties Orthogonal transformations: definition and properties
7. Э.Б. Винберг. Курс алгебры, любое издание, напр., М.: МЦНМО, 2011. URL:http://mathprofi.com/uploads/files/2581_f_41_e.b.vinberg-kurs-algebry-2-e-izd.pdf?key=d04a1718e76a1b8366c8fc0d4d87caf3 (free)	Systems of linear algebraic equations. Cramer's rule. Rouche–Capelli theorem. General solution of a system of linear equations Vector spaces. Basis, dimension of a vector space. Subspaces. Sum and intersection of vector subspaces. Linear transformation of a finite-dimensional vector space, the matrix of a linear transformation. Changing the matrix of a linear transformation with the change of basis Eigenvectors and eigenvalues, their properties Euclidean spaces. Linear transformations on Euclidean spaces Bilinear and quadratic forms

Sources in English	**Corresponding topic**
1. A. C. Burdette. Analytic Geometry. Academic Press, 1971. URL:https://www.amazon.com/Analytic-Geometry-C-Burdette-ebook/dp/B01DUEBGW8 (not free) URL:https://www.amazon.com/Introduction-Analytic-Geometry-Calculus/dp/B00201CRGW (not free)	Vectors and their properties. Dot (scalar), cross (vector) and triple scalar (mixed) products Ways to define a line and a plane. Calculating the angle between a plane and a line, and between two lines. Calculating the distance from a point to a line and the distance between two lines Second-order curves (conic sections). Ellipse, parabola, hyperbola Second-order surfaces (quadric surfaces). Ellipsoid, one-sheeted hyperboloid, two-sheeted hyperboloid, elliptic paraboloid, hyperbolic paraboloid, cone Affine transformations: definition and properties Orthogonal transformations: definition and properties

2. B. Spain. Analytical Geometry. Pergamon, 1963. URL:https://download.tuxfamily.org/openmathdep/geometry_analytic/Analytical_Geometry-Spain.pdf (free)	Vectors and their properties. Dot (scalar), cross (vector) and triple scalar (mixed) products Ways to define a line and a plane. Calculating the angle between a plane and a line, and between two lines. Calculating the distance from a point to a line and the distance between two lines Second-order curves (conic sections). Ellipse, parabola, hyperbola Second-order surfaces (quadric surfaces). Ellipsoid, one-sheeted hyperboloid, two-sheeted hyperboloid, elliptic paraboloid, hyperbolic paraboloid, cone Affine transformations: definition and properties Orthogonal transformations: definition and properties
3. I. M. Gel'fand. Lectures on Linear Algebra. Dover Publications, 1989. URL:https://www.amazon.com/Lectures-Linear-Algebra-Dover-Mathematics/dp/0486660826 (not free)	Systems of linear algebraic equations. Cramer's rule. Rouche–Capelli theorem. General solution of a system of linear equations Vector spaces. Basis, dimension of a vector space. Subspaces. Sum and intersection of vector subspaces. Linear transformation of a finite-dimensional vector space, the matrix of a linear transformation. Changing the matrix of a linear transformation with the change of basis Eigenvectors and eigenvalues, their properties Euclidean spaces. Linear transformations on Euclidean spaces Bilinear and quadratic forms
4. R. Bronson, J.T. Saccoman, G. Costa. Linear Algebra: introduction. Academic Press, 2013. URL:https://mathematicalolympiads.files.wordpress.com/2012/08/lineer-algebra2.pdf (free)	Systems of linear algebraic equations. Cramer's rule. Rouche–Capelli theorem. General solution of a system of linear equations Vector spaces. Basis, dimension of a vector space. Subspaces. Sum and intersection of vector subspaces. Linear transformation of a finite-dimensional vector space, the matrix of a linear transformation. Changing the matrix of a linear transformation with the change of basis Eigenvectors and eigenvalues, their properties Euclidean spaces. Linear transformations on Euclidean spaces Bilinear and quadratic forms
5. S. Axler. Linear Algebra Done Right. Springer, 2015.	Systems of linear algebraic equations. Cramer's rule. Rouche–Capelli theorem.

URL:https://www.amazon.com/Linear-Algebra-Right-Undergraduate-Mathematics/dp/0387982582 (not free) URL:https://ochicken.top/Library/Mathematics/Linear_Algebra/(Undergraduate%20texts%20in%20mathematics)%20Sheldon%20Axler%20-%20Linear%20Algebra%20Done%20Right-Springer%20(1997).pdf (free)	General solution of a system of linear equations Vector spaces. Basis, dimension of a vector space. Subspaces. Sum and intersection of vector subspaces. Linear transformation of a finite-dimensional vector space, the matrix of a linear transformation. Changing the matrix of a linear transformation with the change of basis Eigenvectors and eigenvalues, their properties Euclidean spaces. Linear transformations on Euclidean spaces Bilinear and quadratic forms
6. S. Andrilli, D.Hecker. Elementary Linear Algebra. Academic Press, 2016. URL:https://www.amazon.com/Elementary-Linear-Algebra-Stephen-Andrilli/dp/0123747511 (not free)	Systems of linear algebraic equations. Cramer's rule. Rouche–Capelli theorem. General solution of a system of linear equations Vector spaces. Basis, dimension of a vector space. Subspaces. Sum and intersection of vector subspaces. Linear transformation of a finite-dimensional vector space, the matrix of a linear transformation. Changing the matrix of a linear transformation with the change of basis Eigenvectors and eigenvalues, their properties Euclidean spaces. Linear transformations on Euclidean spaces Bilinear and quadratic forms
7. V. A. Ilyin, E. G. Poznyak . Linear Algebra. Collets, 1986. URL:https://urss.ru/cgi-bin/db.pl (not free) https://www.amazon.com/Linear-Algebra-V-Ilyin/dp/0828533407 (not free)	Systems of linear algebraic equations. Cramer's rule. Rouche–Capelli theorem. General solution of a system of linear equations Vector spaces. Basis, dimension of a vector space. Subspaces. Sum and intersection of vector subspaces. Linear transformation of a finite-dimensional vector space, the matrix of a linear transformation. Changing the matrix of a linear transformation with the change of basis Eigenvectors and eigenvalues, their properties Euclidean spaces. Linear transformations on Euclidean spaces Bilinear and quadratic forms
8. W.R. Gondin, B. Sohmer. Intermediate Algebra & Analytic Geometry. Made Simple, 1965. URL:https://www.amazon.com/Intermediate-Algebra-Analytic-Geometry-William/dp/1483256707 (not free)	Systems of linear algebraic equations. Cramer's rule. Rouche–Capelli theorem. General solution of a system of linear equations Vector spaces. Basis, dimension of a vector space. Subspaces. Sum and intersection of

	vector subspaces. Linear transformation of a finite-dimensional vector space, the matrix of a linear transformation. Changing the matrix of a linear transformation with the change of basis Eigenvectors and eigenvalues, their properties Euclidean spaces. Linear transformations on Euclidean spaces Bilinear and quadratic forms Vectors and their properties. Dot (scalar), cross (vector) and triple scalar (mixed) products Ways to define a line and a plane. Calculating the angle between a plane and a line, and between two lines. Calculating the distance from a point to a line and the distance between two lines Second-order curves (conic sections). Ellipse, parabola, hyperbola Second-order surfaces (quadric surfaces). Ellipsoid, one-sheeted hyperboloid, two-sheeted hyperboloid, elliptic paraboloid, hyperbolic paraboloid, cone Affine transformations: definition and properties Orthogonal transformations: definition and properties
9. Ya. S. Bugrov, S.M. Nikolsky. Fundamentals of Linear Algebra and Analytical Geometry. Mir, 1982. URL:https://vdocuments.net/mir-bugrov-y-s-and-nikolsky-s-m-fundamentals-of-linear-algebra-and.html?page=3 (free)	Systems of linear algebraic equations. Cramer's rule. Rouche–Capelli theorem. General solution of a system of linear equations Vector spaces. Basis, dimension of a vector space. Subspaces. Sum and intersection of vector subspaces. Linear transformation of a finite-dimensional vector space, the matrix of a linear transformation. Changing the matrix of a linear transformation with the change of basis Eigenvectors and eigenvalues, their properties Euclidean spaces. Linear transformations on Euclidean spaces Bilinear and quadratic forms Vectors and their properties. Dot (scalar), cross (vector) and triple scalar (mixed) products Ways to define a line and a plane. Calculating the angle between a plane and a line, and between two lines. Calculating the distance from a point to a line and the distance between two lines

	Second-order curves (conic sections). Ellipse, parabola, hyperbola Second-order surfaces (quadric surfaces). Ellipsoid, one-sheeted hyperboloid, two-sheeted hyperboloid, elliptic paraboloid, hyperbolic paraboloid, cone Affine transformations: definition and properties Orthogonal transformations: definition and properties

Section 2. Real and complex analysis (calculus)

Sources in Russian	**Corresponding topic**
1. А.Г. Свешников, А. Н. Тихонов. Теория функции комплексного переменного, любое издание, напр., М. Физматлит, 2005. URL:http://read.newlibrary.ru/read.php/pdf=15234 (free)	Complex numbers: definition, properties Analytic functions Integral of a function of a complex variable. Cauchy's integral theorem. Cauchy's integral formula Maximum modulus principle and the Schwarz lemma The Taylor and Laurent series Residues of complex functions. Cauchy's residue theorem. Calculation of integrals using residues. Jordan's lemma Conformal mappings. Möbius transformations. The Joukowsky transform
2. А. И. Маркушевич. Краткий курс теории аналитических функций, любое издание, напр., М.: Мир, 2006. URL:https://obuchalka.org/20200827124103/kratkii-kurs-teorii-analiticheskih-funkcii-markushevich-a-i.html (free)	Complex numbers: definition, properties Analytic functions Integral of a function of a complex variable. Cauchy's integral theorem. Cauchy's integral formula Maximum modulus principle and the Schwarz lemma The Taylor and Laurent series Residues of complex functions. Cauchy's residue theorem. Calculation of integrals using residues. Jordan's lemma Conformal mappings. Möbius transformations. The Joukowsky transform
3. И. И. Привалов. Введение в теорию функции комплексного переменного, любое издание, напр., М.: Наука, 1984.	Complex numbers: definition, properties Analytic functions

URL:http://elibrary.sgu.ru/uch_lit/559.pdf (free)	Integral of a function of a complex variable. Cauchy's integral theorem. Cauchy's integral formula Maximum modulus principle and the Schwarz lemma The Taylor and Laurent series Residues of complex functions. Cauchy's residue theorem. Calculation of integrals using residues. Jordan's lemma Conformal mappings. Möbius transformations. The Joukowsky transform
4. С.А. Теляковский. Курс лекций по математическому анализу, семестры 1, 2, 3. М.: МИАН, 2009, 2011, 2013. URL:http://www.mathnet.ru/links/c94dc3095bc0709f9821e3986416a9d8/lkn11.pdf (сем. 1) (free) URL:http://www.mathnet.ru/links/b07162e094c98d486a21ca0069e0cf08/lkn17.pdf (сем. 2) (free) URL:http://www.mathnet.ru/links/094a6c684e3207db229087e38dab9dbf/lkn20.pdf (сем. 3) (free)	Limit of a sequence: definition and properties. Cauchy's criterion. Limit inferior and limit superior. The Bolzano–Weierstrass theorem. The limit of a function at a point. The equivalence of Cauchy's and Heine's definitions Function continuity at a point. Behavior of a continuous function on a segment: the Weierstrass and Bolzano-Cauchy theorems. The inverse function theorem. Uniform continuity, Cantor's theorem. Derivative of a function (of a single variable) at a point: definition and basic properties. The derivative of a composition of functions. Differentiability of a function at a point. Function differential at a point. Derivative of an inverse function. Higher order derivatives and differentials. The Leibniz rule. Rolle's Theorem, the mean value theorem, Cauchy's mean value theorem. L'Hôpital's rule. Taylor's expansion with the Peano and Lagrange form of the remainder. Using Taylor's expansion and L'Hôpital's rule for limit calculations. Using derivative to study the properties of a function of a single variable: monotonicity, extrema, convexity, inflection points Indefinite integral. Definite integral. Darboux's criterion of the integrability of a function. Properties of an integral with a variable upper limit: continuity, differentiability. The Newton–Leibniz formula. Geometric applications of definite integral. Improper integrals. Absolute

	convergence and conditional convergence. Cauchy's criterion, Dirichlet's test of convergence Differentiability of a function of several variables. Necessary conditions and sufficient conditions for differentiability. Gradient of a function. Implicit function theorem. Local extrema of a function of several variables. Necessary conditions and sufficient conditions of a local extremum point. Conditional extrema of a function. The Langrange multiplier method, necessary and sufficient conditions for conditional extrema Numeric series. Absolute and conditional convergence. Cauchy's criterion, comparison test, integral test, the Root and Ratio tests, Leibniz's and Dirichlet's tests. Function series. Uniform convergence. Cauchy's criterion, the Weierstrass test, Dirichlet's test of uniform convergence. Power series. The radius of convergence, the Cauchy—Hadamard formula. The Taylor expansion. Taylor expansions for elementary functions Line integral. Green's theorem. Surface integrals. The Gauss—Ostrogradsky theorem. Stokes' theorem A Fourier series. Pointwise convergence conditions. Uniform convergence conditions. The Fourier transform of an absolutely integrable function and its properties. The Fourier transform of a derivative and the derivative of a Fourier transform
5. С. М. Никольский. Курс математического анализа. В 2-х томах. М.: Физматлит,2001. URL:http://www.tka4.org/materials/lib/Articles-Books/General/MathAnalysis/NIKOLSKI1.PDF (free) URL:http://www.physics.gov.az/book_K/NIKOLSKI2.PDF (free)	Limit of a sequence: definition and properties. Cauchy's criterion. Limit inferior and limit superior. The Bolzano–Weierstrass theorem. The limit of a function at a point. The equivalence of Cauchy's and Heine's definitions Function continuity at a point. Behavior of a continuous function on a segment: the Weierstrass and Bolzano-Cauchy theorems. The inverse function theorem. Uniform continuity, Cantor's theorem. Derivative of a function (of a single variable) at a point: definition and basic properties. The

	derivative of a composition of functions. Differentiability of a function at a point. Function differential at a point. Derivative of an inverse function. Higher order derivatives and differentials. The Leibniz rule. Rolle's Theorem, the mean value theorem, Cauchy's mean value theorem. L'Hôpital's rule. Taylor's expansion with the Peano and Lagrange form of the remainder. Using Taylor's expansion and L'Hôpital's rule for limit calculations. Using derivative to study the properties of a function of a single variable: monotonicity, extrema, convexity, inflection points Indefinite integral. Definite integral. Darboux's criterion of the integrability of a function. Properties of an integral with a variable upper limit: continuity, differentiability. The Newton–Leibniz formula. Geometric applications of definite integral. Improper integrals. Absolute convergence and conditional convergence. Cauchy's criterion, Dirichlet's test of convergence Differentiability of a function of several variables. Necessary conditions and sufficient conditions for differentiability. Gradient of a function. Implicit function theorem. Local extrema of a function of several variables. Necessary conditions and sufficient conditions of a local extremum point. Conditional extrema of a function. The Langrange multiplier method, necessary and sufficient conditions for conditional extrema Numeric series. Absolute and conditional convergence. Cauchy's criterion, comparison test, integral test, the Root and Ratio tests, Leibniz's and Dirichlet's tests. Function series. Uniform convergence. Cauchy's criterion, the Weierstrass test, Dirichlet's test of uniform convergence. Power series. The radius of convergence, the Cauchy—Hadamard formula. The Taylor expansion. Taylor expansions for elementary functions Line integral. Green's theorem. Surface integrals. The Gauss—Ostrogradsky theorem. Stokes' theorem

	A Fourier series. Pointwise convergence conditions. Uniform convergence conditions. The Fourier transform of an absolutely integrable function and its properties. The Fourier transform of a derivative and the derivative of a Fourier transform
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	Conformal mappings. Möbius transformations. The Joukowsky transform
2. D.E. Marshall. Complex Analysis. Cambridge University Press, 2019. URL:https://www.matem.unam.mx/~hector/[Lars_Ahlfors]_Complex_Analysis_(Third_Edition).pdf (free) URL:https://www.amazon.com/Complex-Analysis-Cambridge-Mathematical-Textbooks/dp/110713482X (not free)	Complex numbers: definition, properties Analytic functions Integral of a function of a complex variable. Cauchy's integral theorem. Cauchy's integral formula Maximum modulus principle and the Schwarz lemma The Taylor and Laurent series Residues of complex functions. Cauchy's residue theorem. Calculation of integrals using residues. Jordan's lemma Conformal mappings. Möbius transformations. The Joukowsky transform
3. G. Strang. Calculus. 3rd edition. Wellesley-Cambridge Press, 2017. URL:https://ocw.mit.edu/ans7870/resources/Strang/Edited/Calculus/Calculus.pdf (free) URL:https://www.amazon.com/Calculus-3rd-Gilbert-Strang/dp/0980232759 (not free)	Limit of a sequence: definition and properties. Cauchy's criterion. Limit inferior and limit superior. The Bolzano–Weierstrass theorem. The limit of a function at a point. The equivalence of Cauchy's and Heine's definitions Function continuity at a point. Behavior of a continuous function on a segment: the Weierstrass and Bolzano-Cauchy theorems. The inverse function theorem. Uniform continuity, Cantor's theorem. Derivative of a function (of a single variable) at a point: definition and basic properties. The derivative of a composition of functions. Differentiability of a function at a point. Function differential at a point. Derivative of an inverse function. Higher order derivatives and differentials. The Leibniz rule. Rolle's Theorem, the mean value theorem, Cauchy's mean value theorem. L'Hôpital's rule. Taylor's expansion with the Peano and Lagrange form of the remainder. Using Taylor's expansion and L'Hôpital's rule for limit calculations. Using derivative to study the properties of a function of a single variable: monotonicity, extrema, convexity, inflection points Indefinite integral. Definite integral. Darboux's criterion of the integrability of a

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4. H. Flanders, R.R. Korfhage, J.J. Price. A Second Course in Calculus. Academic Press, 1974. URL:https://www.amazon.com/Second-Course-Calculus-Harley-Flanders-ebook/dp/B01DUEGPE2 (not free)	Limit of a sequence: definition and properties. Cauchy's criterion. Limit inferior and limit superior. The Bolzano–Weierstrass theorem. The limit of a function at a point. The equivalence of Cauchy's and Heine's definitions Function continuity at a point. Behavior of a continuous function on a segment: the Weierstrass and Bolzano-Cauchy theorems.

	The inverse function theorem. Uniform continuity, Cantor's theorem. Derivative of a function (of a single variable) at a point: definition and basic properties. The derivative of a composition of functions. Differentiability of a function at a point. Function differential at a point. Derivative of an inverse function. Higher order derivatives and differentials. The Leibniz rule. Rolle's Theorem, the mean value theorem, Cauchy's mean value theorem. L'Hôpital's rule. Taylor's expansion with the Peano and Lagrange form of the remainder. Using Taylor's expansion and L'Hôpital's rule for limit calculations. Using derivative to study the properties of a function of a single variable: monotonicity, extrema, convexity, inflection points Indefinite integral. Definite integral. Darboux's criterion of the integrability of a function. Properties of an integral with a variable upper limit: continuity, differentiability. The Newton–Leibniz formula. Geometric applications of definite integral. Improper integrals. Absolute convergence and conditional convergence. Cauchy's criterion, Dirichlet's test of convergence Differentiability of a function of several variables. Necessary conditions and sufficient conditions for differentiability. Gradient of a function. Implicit function theorem. Local extrema of a function of several variables. Necessary conditions and sufficient conditions of a local extremum point. Conditional extrema of a function. The Langrange multiplier method, necessary and sufficient conditions for conditional extrema Numeric series. Absolute and conditional convergence. Cauchy's criterion, comparison test, integral test, the Root and Ratio tests, Leibniz's and Dirichlet's tests. Function series. Uniform convergence. Cauchy's criterion, the Weierstrass test, Dirichlet's test of uniform convergence. Power series. The radius of convergence, the Cauchy—Hadamard

	formula. The Taylor expansion. Taylor expansions for elementary functions Line integral. Green's theorem. Surface integrals. The Gauss—Ostrogradsky theorem. Stokes' theorem A Fourier series. Pointwise convergence conditions. Uniform convergence conditions. The Fourier transform of an absolutely integrable function and its properties. The Fourier transform of a derivative and the derivative of a Fourier transform
5. L. Ahlfors, L. V., Complex analysis. McGraw-Hill, 1986. URL:https://www.matem.unam.mx/~hector/[Lars_Ahlfors]_Complex_Analysis_(Third_Edition).pdf (free) URL:https://www.amazon.com/Complex-Analysis-Lars-Ahlfors/dp/0070006571 (not free)	Complex numbers: definition, properties Analytic functions Integral of a function of a complex variable. Cauchy's integral theorem. Cauchy's integral formula Maximum modulus principle and the Schwarz lemma The Taylor and Laurent series Residues of complex functions. Cauchy's residue theorem. Calculation of integrals using residues. Jordan's lemma Conformal mappings. Möbius transformations. The Joukowsky transform
6. L. Loomis, S. Steinberg. Advanced Calculus. Revised edition, Londom-Boston, 1990 URL:https://people.math.harvard.edu/~shlomo/docs/Advanced_Calculus.pdf (free)	Limit of a sequence: definition and properties. Cauchy's criterion. Limit inferior and limit superior. The Bolzano–Weierstrass theorem. The limit of a function at a point. The equivalence of Cauchy's and Heine's definitions Function continuity at a point. Behavior of a continuous function on a segment: the Weierstrass and Bolzano-Cauchy theorems. The inverse function theorem. Uniform continuity, Cantor's theorem. Derivative of a function (of a single variable) at a point: definition and basic properties. The derivative of a composition of functions. Differentiability of a function at a point. Function differential at a point. Derivative of an inverse function. Higher order derivatives and differentials. The Leibniz rule. Rolle's Theorem, the mean value theorem, Cauchy's mean value theorem. L'Hôpital's rule. Taylor's expansion with the Peano and

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Functions of a Complex Variable. Mir, 1985. URL:https://www.amazon.com/Lectures-Functions-Variable-I-Shabunin-Facsimile/dp/B07L35NPHT (not free) URL:http://theory.fi.infn.it/colomo/metodi/testi/Sidorov_Fedoryuk_Shabunin.pdf (free)	Integral of a function of a complex variable. Cauchy's integral theorem. Cauchy's integral formula Maximum modulus principle and the Schwarz lemma The Taylor and Laurent series Residues of complex functions. Cauchy's residue theorem. Calculation of integrals using residues. Jordan's lemma Conformal mappings. Möbius transformations. The Joukowsky transform
8. Reiner Kuhnau. Handbook of Complex Analysis. North Holland, 2004. URL:https://www.amazon.com/Handbook-Complex-Analysis-Reiner-Kuhnau/dp/0444828451 (not free)	Complex numbers: definition, properties Analytic functions Integral of a function of a complex variable. Cauchy's integral theorem. Cauchy's integral formula Maximum modulus principle and the Schwarz lemma The Taylor and Laurent series Residues of complex functions. Cauchy's residue theorem. Calculation of integrals using residues. Jordan's lemma Conformal mappings. Möbius transformations. The Joukowsky transform
9. T. Dence, J. Dence. Advanced Calculus. Academic Press, 2009. URL:https://www.amazon.com/Advanced-Calculus-Transition-Thomas-Dence/dp/0123749557 (not free)	Limit of a sequence: definition and properties. Cauchy's criterion. Limit inferior and limit superior. The Bolzano–Weierstrass theorem. The limit of a function at a point. The equivalence of Cauchy's and Heine's definitions Function continuity at a point. Behavior of a continuous function on a segment: the Weierstrass and Bolzano-Cauchy theorems. The inverse function theorem. Uniform continuity, Cantor's theorem. Derivative of a function (of a single variable) at a point: definition and basic properties. The derivative of a composition of functions. Differentiability of a function at a point. Function differential at a point. Derivative of an inverse function. Higher order derivatives and differentials. The Leibniz rule. Rolle's Theorem, the mean value theorem, Cauchy's mean value theorem. L'Hôpital's rule.

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10. Ya. S. Bugrov, S. M. Nikolsky. Differential and Integral Calculus. Imported Pubn, 1983. URL:https://www.amazon.com/Differential-Integral-Calculus-Ya-Bugrov/dp/0828523061 (not free)	Limit of a sequence: definition and properties. Cauchy's criterion. Limit inferior and limit superior. The Bolzano–Weierstrass theorem. The limit of a function at a point. The equivalence of Cauchy's and Heine's definitions Function continuity at a point. Behavior of a continuous function on a segment: the Weierstrass and Bolzano-Cauchy theorems. The inverse function theorem. Uniform continuity, Cantor's theorem. Derivative of a function (of a single variable) at a point: definition and basic properties. The derivative of a composition of functions. Differentiability of a function at a point. Function differential at a point. Derivative of an inverse function. Higher order derivatives and differentials. The Leibniz rule. Rolle's Theorem, the mean value theorem, Cauchy's mean value theorem. L'Hôpital's rule. Taylor's expansion with the Peano and Lagrange form of the remainder. Using Taylor's expansion and L'Hôpital's rule for limit calculations. Using derivative to study the properties of a function of a single variable: monotonicity, extrema, convexity, inflection points Indefinite integral. Definite integral. Darboux's criterion of the integrability of a function. Properties of an integral with a variable upper limit: continuity, differentiability. The Newton–Leibniz formula. Geometric applications of definite integral. Improper integrals. Absolute convergence and conditional convergence. Cauchy's criterion, Dirichlet's test of convergence Differentiability of a function of several variables. Necessary conditions and sufficient conditions for differentiability. Gradient of a function. Implicit function theorem. Local extrema of a function of several variables. Necessary conditions and sufficient conditions of a local extremum point. Conditional extrema of a function. The Langrange multiplier method, necessary and sufficient conditions for conditional extrema

	Numeric series. Absolute and conditional convergence. Cauchy's criterion, comparison test, integral test, the Root and Ratio tests, Leibniz's and Dirichlet's tests. Function series. Uniform convergence. Cauchy's criterion, the Weierstrass test, Dirichlet's test of uniform convergence. Power series. The radius of convergence, the Cauchy—Hadamard formula. The Taylor expansion. Taylor expansions for elementary functions Line integral. Green's theorem. Surface integrals. The Gauss—Ostrogradsky theorem. Stokes' theorem A Fourier series. Pointwise convergence conditions. Uniform convergence conditions. The Fourier transform of an absolutely integrable function and its properties. The Fourier transform of a derivative and the derivative of a Fourier transform

Section 3. Differential equations

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2. А.П. Карташев, Б.Л. Рождественский. Обыкновенные дифференциальные уравнения и основы вариационного исчисления. М.: Наука, 1980. URL:https://obuchalka.org/20130402 70515/obiknovennie-differencialnie-	Ordinary differential equations. Separation of variables. Reduction of order of the differential equation Linear ODEs and systems of ODEs with constant coefficients Linear ODEs and systems of ODEs with variable coefficients. A fundamental solution

uravneniya-i-osnovi-variacionnogo-ischisleniya-kartashev-a-p-rojdestvenskii-b-l-1980.html (free)	set. The Wrońskian, Liouville's formula. The variation of constants method Equilibrium of an autonomous system of differential equations. Classifying the equilibria of linear autonomous second-order systems. Stability and asymptotic stability of equilibrium. First integrals of autonomous system of differential equations. Theorem on the number of independent first integrals Calculus of variations. The Euler–Lagrange equation. Necessary condition for a weak local extremum
3. А.Ф. Филлипов. Введение в теорию дифференциальных уравнений. М.: КомКнига, 2007. URL:https://obuchalka.org/20140323 76465/vvedenie-v-teoriu-differencialnih-uravnenii-filippov-a-f-2007.html (free)	Ordinary differential equations. Separation of variables. Reduction of order of the differential equation Linear ODEs and systems of ODEs with constant coefficients Linear ODEs and systems of ODEs with variable coefficients. A fundamental solution set. The Wrońskian, Liouville's formula. The variation of constants method Equilibrium of an autonomous system of differential equations. Classifying the equilibria of linear autonomous second-order systems. Stability and asymptotic stability of equilibrium. First integrals of autonomous system of differential equations. Theorem on the number of independent first integrals
4. А.Ф. Филиппов. Сборник задач по дифференциальным уравнениям. Ижевск: НИЦ "Регулярная и хаотическая динамика", 2000. URL:http://kvm.gubkin.ru/pub/uok/FilippovDU.pdf (free)	Ordinary differential equations. Separation of variables. Reduction of order of the differential equation Linear ODEs and systems of ODEs with constant coefficients Linear ODEs and systems of ODEs with variable coefficients. A fundamental solution set. The Wrońskian, Liouville's formula. The variation of constants method Equilibrium of an autonomous system of differential equations. Classifying the equilibria of linear autonomous second-order systems. Stability and asymptotic stability of equilibrium. First integrals of autonomous system of differential equations. Theorem on the number of independent first integrals
5. Л.Э. Эльсгольц. Дифференциальные уравнения и вариационное исчисление. М.: Наука, 1965.	Ordinary differential equations. Separation of variables. Reduction of order of the differential equation

URL:http://www.phys.nsu.ru/balakina/El%27sgol%27dz_Dif_ur_i_var_isch.pdf (free)	Linear ODEs and systems of ODEs with constant coefficients Linear ODEs and systems of ODEs with variable coefficients. A fundamental solution set. The Wrońskian, Liouville's formula. The variation of constants method Equilibrium of an autonomous system of differential equations. Classifying the equilibria of linear autonomous second-order systems. Stability and asymptotic stability of equilibrium. First integrals of autonomous system of differential equations. Theorem on the number of independent first integrals Calculus of variations. The Euler–Lagrange equation. Necessary condition for a weak local extremum
6. М.В. Федорюк. Обыкновенные дифференциальные уравнения. М.: Наука, 1985. URL:http://cmcstuff.esyr.org/vmkbotva-r15/2%20%D0%BA%D1%83%D1%80%D1%81/4%20%D0%A1%D0%B5%D0%BC%D0%B5%D1%81%D1%82%D1%80/%D0%94%D0%B8%D1%84%D1%84%D1%83%D1%80%D1%8B/%D0%A3%D1%87%D0%B5%D0%B1%D0%BD%D0%B8%D0%BA%D0%B8/%D0%A4%D0%B5%D0%B4%D0%BE%D1%80%D1%8E%D0%BA%20%D0%9C.,%20%D0%9E%D0%B1%D1%8B%D0%BA%D0%BD%D0%BE%D0%B2%D0%B5%D0%BD%D0%BD%D1%8B%D0%B5%20%D0%B4%D0%B8%D1%84%D1%84.%D1%83%D1%80%D0%B0%D0%B2%D0%BD%D0%B5%D0%BD%D0%B8%D1%8F,%201985.pdf (free)	Ordinary differential equations. Separation of variables. Reduction of order of the differential equation Linear ODEs and systems of ODEs with constant coefficients Linear ODEs and systems of ODEs with variable coefficients. A fundamental solution set. The Wrońskian, Liouville's formula. The variation of constants method Equilibrium of an autonomous system of differential equations. Classifying the equilibria of linear autonomous second-order systems. Stability and asymptotic stability of equilibrium. First integrals of autonomous system of differential equations. Theorem on the number of independent first integrals
7. Н.М. Матвеев. Методы интегрирования обыкновенных дифференциальных уравнений. Учебник. Минск: «Вышэйшая школа», 1974. URL:https://obuchalka.org/2015033083660/metodi-integrirovaniya-obiknovennih-differencialnih-	Ordinary differential equations. Separation of variables. Reduction of order of the differential equation Linear ODEs and systems of ODEs with constant coefficients Linear ODEs and systems of ODEs with variable coefficients. A fundamental solution

uravnenii-matveev-n-m-1967.html (free) URL:https://cdnpdf.com/pdf-12290-metody-integrirovanija-obyknovennyh-differencialnyh-uravnenij-matveev-nm (free)	set. The Wrońskian, Liouville's formula. The variation of constants method Equilibrium of an autonomous system of differential equations. Classifying the equilibria of linear autonomous second-order systems. Stability and asymptotic stability of equilibrium. First integrals of autonomous system of differential equations. Theorem on the number of independent first integrals

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2. G. Simmons. Differential equations with applications and historical notes. McGraw-Hill, New York, 1991. URL:https://caisatech.net/uploads/120220%20Differential%20equations.pdf (free)	Ordinary differential equations. Separation of variables. Reduction of order of the differential equation Linear ODEs and systems of ODEs with constant coefficients Linear ODEs and systems of ODEs with variable coefficients. A fundamental solution set. The Wrońskian, Liouville's formula. The variation of constants method Equilibrium of an autonomous system of differential equations. Classifying the equilibria of linear autonomous second-order systems. Stability and asymptotic stability of equilibrium. First integrals of autonomous system of differential equations. Theorem on the number of independent first integrals
3. G. Strang. Differential equations and Linear algebra. Wellesley-Cambridge Press, 2014. URL:https://www.amazon.com/Differential-Equations-Linear-Algebra-Gilbert/dp/0980232791#:~:text=Differential%20equations%20and%20linear%20algebra%20are%20two%20central%20topics%20in,giving%20increase	Ordinary differential equations. Separation of variables. Reduction of order of the differential equation Linear ODEs and systems of ODEs with constant coefficients Linear ODEs and systems of ODEs with variable coefficients. A fundamental solution set. The Wrońskian, Liouville's formula. The variation of constants method

d%20flexibility%20to%20instructors (not free) URL:https://math.mit.edu/~gs/dela/ (free)	Equilibrium of an autonomous system of differential equations. Classifying the equilibria of linear autonomous second-order systems. Stability and asymptotic stability of equilibrium. First integrals of autonomous system of differential equations. Theorem on the number of independent first integrals
4. L. Elsgolts. Differential Equations and the Calculus of Variations. University Press of the Pacific , 2003. URL:https://ia800908.us.archive.org/2/items/ElsgoltsDifferentialEquationsAndTheCalculusOfVariations/Elsgolts-Differential-Equations-and-the-Calculus-of-Variations.pdf (free) URL:https://www.amazon.com/Differential-Equations-Calculus-Variations-Elsgolts/dp/1410210677 (not free)	Ordinary differential equations. Separation of variables. Reduction of order of the differential equation Linear ODEs and systems of ODEs with constant coefficients Linear ODEs and systems of ODEs with variable coefficients. A fundamental solution set. The Wrońskian, Liouville's formula. The variation of constants method Equilibrium of an autonomous system of differential equations. Classifying the equilibria of linear autonomous second-order systems. Stability and asymptotic stability of equilibrium. First integrals of autonomous system of differential equations. Theorem on the number of independent first integrals Calculus of variations. The Euler–Lagrange equation. Necessary condition for a weak local extremum
5. M. Tenenbaum, H. Pollard. Ordinary Differential Equations. Dover Publications, 1985. URL:https://netsanet4all.files.wordpress.com/2018/12/294222977-ordinary-differential-equations-tenenbaum-pollard-0486649407.pdf (free)	Ordinary differential equations. Separation of variables. Reduction of order of the differential equation Linear ODEs and systems of ODEs with constant coefficients Linear ODEs and systems of ODEs with variable coefficients. A fundamental solution set. The Wrońskian, Liouville's formula. The variation of constants method Equilibrium of an autonomous system of differential equations. Classifying the equilibria of linear autonomous second-order systems. Stability and asymptotic stability of equilibrium. First integrals of autonomous system of differential equations. Theorem on the number of independent first integrals

Section 4. Theory of probability and mathematical statistics

Sources in Russian	Corresponding topic
1. Б.В. Гнеденко. Курс теории вероятностей. 8-е изд., испр. и доп.—М.: Едиториал УРСС, 2005. URL:https://obuchalka.org/20190716111540/kurs-teorii-veroyatnostei-uchebnik-gnedenko-b-v-2011.html (free)	Fundamentals of combinatorics: counting rules, addition and multiplication rules, combinations with and without repetition, binomial coefficients and Newton's binomial theorem Probability space. Independent events. Summation theorem. Conditional probability. Collectively exhaustive events. Law of total probability. Bayes' formula A random variable and its cumulative distribution function. Expectation and variance of a random variable: definition and properties Basic classes of probability distributions: binomial, geometric, uniform, Poisson, exponential, normal. Bernoulli trials. Chebyshev's inequality. Law of large numbers Joint probability distributions. Independence. Covariance. Correlation coefficient
2. В. П. Чистяков. Курс теории вероятностей, любое издание URL:https://obuchalka.org/20210219129486/kurs-teorii-veroyatnostei-chistyakov-v-p-2000.html (free) URL:https://vk.com/doc409016625_585522734?hash=shU94s8Mkzaz5HeKr1ywztg9SrhlDNtiQACwqblc5zk&dl=56IpiYMTeuz3eGbp6mbOGSR1yNcyUBZ80zEA05ykrbL (free)	Fundamentals of combinatorics: counting rules, addition and multiplication rules, combinations with and without repetition, binomial coefficients and Newton's binomial theorem Probability space. Independent events. Summation theorem. Conditional probability. Collectively exhaustive events. Law of total probability. Bayes' formula A random variable and its cumulative distribution function. Expectation and variance of a random variable: definition and properties Basic classes of probability distributions: binomial, geometric, uniform, Poisson, exponential, normal. Bernoulli trials. Chebyshev's inequality. Law of large numbers Joint probability distributions. Independence. Covariance. Correlation coefficient
3. В. Феллер. Введение в теорию вероятностей и ее приложения, любое издание. URL:https://www.labirint.ru/books/828835/ (not free) URL:https://vk.com/doc409016625_541696520?hash=WCRmDd8Eiwo6wg	Fundamentals of combinatorics: counting rules, addition and multiplication rules, combinations with and without repetition, binomial coefficients and Newton's binomial theorem Probability space. Independent events. Summation theorem. Conditional probability.

lbHb3UEgzAKoxswEkGwwRizPtgqKc&dl=OWNlrVPbnWq1foZAAKUq944tkgs20FoTsDXkU9LETI4 (free) URL:https://vk.com/doc409016625_541697907?hash=09ukiWbbZdvhqbsNPXPFZHdAO0TUJSWF3qxvTMHJ5S4&dl=NwYIWvIFGAE8zJ7Qe9vTZ98gXzwDnaaWwtzh5ZdKAwX (free)	Collectively exhaustive events. Law of total probability. Bayes' formula A random variable and its cumulative distribution function. Expectation and variance of a random variable: definition and properties Basic classes of probability distributions: binomial, geometric, uniform, Poisson, exponential, normal. Bernoulli trials. Chebyshev's inequality. Law of large numbers Joint probability distributions. Independence. Covariance. Correlation coefficient
4. Г.И. Ивченко, Ю.И. Медведев. Математическая статистика. М.: Высш. шк., 1984. URL:https://www.hse.ru/pubs/share/direct/content_document/103185710 (free)	Estimating the parameters of a distribution. Statistical hypothesis testing
5. Е. С. Вентцель. Теория вероятностей, 2006 URL:https://obuchalka.org/20190227107251/teoriya-veroyatnostei-ventcel-e-s-2006.html (free)	Fundamentals of combinatorics: counting rules, addition and multiplication rules, combinations with and without repetition, binomial coefficients and Newton's binomial theorem Probability space. Independent events. Summation theorem. Conditional probability. Collectively exhaustive events. Law of total probability. Bayes' formula A random variable and its cumulative distribution function. Expectation and variance of a random variable: definition and properties Basic classes of probability distributions: binomial, geometric, uniform, Poisson, exponential, normal. Bernoulli trials. Chebyshev's inequality. Law of large numbers Joint probability distributions. Independence. Covariance. Correlation coefficient

Sources in English	**Corresponding topic**
1. B.V. Gnedenko. Theory of Probability. CRC Press, 1998. URL:https://www.amazon.com/Theory-Probability-Boris-V-Gnedenko/dp/9056995855 (not free)	Fundamentals of combinatorics: counting rules, addition and multiplication rules, combinations with and without repetition, binomial coefficients and Newton's binomial theorem Probability space. Independent events. Summation theorem. Conditional probability.

	Collectively exhaustive events. Law of total probability. Bayes' formula A random variable and its cumulative distribution function. Expectation and variance of a random variable: definition and properties Basic classes of probability distributions: binomial, geometric, uniform, Poisson, exponential, normal. Bernoulli trials. Chebyshev's inequality. Law of large numbers Joint probability distributions. Independence. Covariance. Correlation coefficient
2. E.L. Lehmann, G. Casella. Theory of Point Estimation (2nd ed.). New York: Springer, 1998. URL:https://www.dcpehvpm.org/E-Content/Stat/E%20L%20Lehaman.pdf (free)	Estimating the parameters of a distribution. Statistical hypothesis testing
3. E.L. Lehmann, Joseph P. Romano. Testing Statistical Hypotheses (3rd ed.). New York: Springer, 2005. URL:https://sites.stat.washington.edu/jaw/COURSES/580s/582/HO/Lehmann_and_Romano-TestingStatisticalHypotheses.pdf (free)	Estimating the parameters of a distribution. Statistical hypothesis testing
4. G.R. Grimmett, D.R. Stirzaker. Probability and Random Processes. Oxford University Press, 2001. URL:http://home.ustc.edu.cn/~zt001062/PTmaterials/Grimmett&Stirzaker--Probability%20and%20Random%20Processes%20%20Third%20Ed(2001).pdf (free)	Fundamentals of combinatorics: counting rules, addition and multiplication rules, combinations with and without repetition, binomial coefficients and Newton's binomial theorem Probability space. Independent events. Summation theorem. Conditional probability. Collectively exhaustive events. Law of total probability. Bayes' formula A random variable and its cumulative distribution function. Expectation and variance of a random variable: definition and properties Basic classes of probability distributions: binomial, geometric, uniform, Poisson, exponential, normal. Bernoulli trials. Chebyshev's inequality. Law of large numbers Joint probability distributions. Independence. Covariance. Correlation coefficient
5. R.E. Walpol, R.H. Myers, S.L. Myers, K. Ye. Probability and Statistics. Prentice Hall, 2011. URL:https://spada.uns.ac.id/pluginfile	Fundamentals of combinatorics: counting rules, addition and multiplication rules, combinations with and without repetition,

.php/221008/mod_resource/content/1/ProbabilityStatistics_for_EngineersScientists%289th_Edition%29_Walpole.pdf (free)	binomial coefficients and Newton's binomial theorem Probability space. Independent events. Summation theorem. Conditional probability. Collectively exhaustive events. Law of total probability. Bayes' formula A random variable and its cumulative distribution function. Expectation and variance of a random variable: definition and properties Basic classes of probability distributions: binomial, geometric, uniform, Poisson, exponential, normal. Bernoulli trials. Chebyshev's inequality. Law of large numbers Joint probability distributions. Independence. Covariance. Correlation coefficient Estimating the parameters of a distribution. Statistical hypothesis testing
6. V.K. Zakharov, V. P. Chistyakov, B. A. Sevastyanov. Probability Theory for Engineers. Optimization Software. 1987. URL:https://www.alibris.com/booksearch.detail?invid=17109690527&isbn=9780911575132&utm_medium=affiliate&utm_source=GuWPtmTDDdQ&utm_campaign=2&siteID=GuWPtmTDDdQ-bX4XKw.oyIGLVgYhZ53VdQ (not free)	Fundamentals of combinatorics: counting rules, addition and multiplication rules, combinations with and without repetition, binomial coefficients and Newton's binomial theorem Probability space. Independent events. Summation theorem. Conditional probability. Collectively exhaustive events. Law of total probability. Bayes' formula A random variable and its cumulative distribution function. Expectation and variance of a random variable: definition and properties Basic classes of probability distributions: binomial, geometric, uniform, Poisson, exponential, normal. Bernoulli trials. Chebyshev's inequality. Law of large numbers Joint probability distributions. Independence. Covariance. Correlation coefficient
7. W. Feller. An Introduction to Probability Theory and its Applications. John Wiley & Sons, 1967. URL:https://www.amazon.com/Introduction-Probability-Theory-Applications-Vol/dp/0471257087 (not free) URL:http://www.ru.ac.bd/stat/wp-content/uploads/sites/25/2019/03/101_06_Feller_An-Introduction-to-	Fundamentals of combinatorics: counting rules, addition and multiplication rules, combinations with and without repetition, binomial coefficients and Newton's binomial theorem Probability space. Independent events. Summation theorem. Conditional probability. Collectively exhaustive events. Law of total probability. Bayes' formula

Probability-Theory-and-Its-Applications-Vol.-2.pdf (free)	A random variable and its cumulative distribution function. Expectation and variance of a random variable: definition and properties Basic classes of probability distributions: binomial, geometric, uniform, Poisson, exponential, normal. Bernoulli trials. Chebyshev's inequality. Law of large numbers Joint probability distributions. Independence. Covariance. Correlation coefficient

Section 5. Machine learning

Sources in Russian	**Corresponding topic**
1. Архангельская Е. Глубокое обучение / С. Николенко, А. Кадурин. –СПБ: Издательский дом "Питер", 2017. URL:https://www.ozon.ru/product/glubokoe-obuchenie-arhangelskaya-e-o-kadurin-a-a-211432536/?sh=dK3WCCOfsA (not free)	Regression analysis. Binary data classification problem. Linear regression model. Multiple linear regression. Ordinary least squares method. Calculating linear regression coefficients. L1 and L2 regularization Regression scoring. Classification scores and margins. Performance of a binary classifier. The confusion matrix. Confusion-matrix-based performance measures. Analysis of the coefficient of determination and the adjusted coefficient of determination. The ROC-curve: calculation, properties, and interpretation. The ROC-curve of a perfect and random classifiers. ROC AUC. The ROC AUC calculation method Hierarchical agglomerative clustering. Dendrogram. Types of metrics in a feature space. Methods for calculating the distance between clusters. Ward's method Dimensionality reduction. Principal component analysis. Independent component analysis
2. Гудфеллоу Я., Иошуа Б., Курвилль А. Глубокое обучение. – Litres, 2018. URL:https://www.ozon.ru/product/glubokoe-obuchenie-tsvetnye-illyustratsii-bendzhio-ioshua-gudfellou-yan-217046706/?sh=dK3WCMy0oA (not free)	Regression analysis. Binary data classification problem. Linear regression model. Multiple linear regression. Ordinary least squares method. Calculating linear regression coefficients. L1 and L2 regularization Metric space. Euclidean space. Taxicab geometry. The k-nearest neighbors algorithm. The weighted k-nearest neighbors algorithm Bayes' theorem. Bayesian decision rule. Naive Bayes classifier

	Decision tree. Decision rules. The Gini impurity. Information gain. Classification and regression trees. Ensemble learning for classification and regression. Bagging. Boosting Regression scoring. Classification scores and margins. Performance of a binary classifier. The confusion matrix. Confusion-matrix-based performance measures. Analysis of the coefficient of determination and the adjusted coefficient of determination. The ROC-curve: calculation, properties, and interpretation. The ROC-curve of a perfect and random classifiers. ROC AUC. The ROC AUC calculation method The mathematical model of an artificial neuron. Activation functions. Feed-forward neural networks. The mathematical model of a multi-layer neural network. Neural network training as an optimization problem. Backpropagation. Calculating the gradient of the objective function. Loss function. Types of loss functions. Comparison of the learning processes with different loss functions. Neural networks training algorithms. Vanilla gradient descent and stochastic gradient descent The k-means method. Problem statement and computational steps (with an example illustration). k-means initialization. The k-means++ algorithm. Choosing the number of clusters. The elbow method Hierarchical agglomerative clustering. Dendrogram. Types of metrics in a feature space. Methods for calculating the distance between clusters. Ward's method Dimensionality reduction. Principal component analysis. Independent component analysis
3. Воронцов К. В. Математические методы обучения по прецедентам (теория обучения машин) // Москва. – 2011. – С. 141. URL:http://www.machinelearning.ru/wiki/images/6/6d/voron-ml-1.pdf (free)	Regression analysis. Binary data classification problem. Linear regression model. Multiple linear regression. Ordinary least squares method. Calculating linear regression coefficients. L1 and L2 regularization Metric space. Euclidean space. Taxicab geometry. The k-nearest neighbors algorithm. The weighted k-nearest neighbors algorithm

	Bayes' theorem. Bayesian decision rule. Naive Bayes classifier Decision tree. Decision rules. The Gini impurity. Information gain. Classification and regression trees. Ensemble learning for classification and regression. Bagging. Boosting Regression scoring. Classification scores and margins. Performance of a binary classifier. The confusion matrix. Confusion-matrix-based performance measures. Analysis of the coefficient of determination and the adjusted coefficient of determination. The ROC-curve: calculation, properties, and interpretation. The ROC-curve of a perfect and random classifiers. ROC AUC. The ROC AUC calculation method The mathematical model of an artificial neuron. Activation functions. Feed-forward neural networks. The mathematical model of a multi-layer neural network. Neural network training as an optimization problem. Backpropagation. Calculating the gradient of the objective function. Loss function. Types of loss functions. Comparison of the learning processes with different loss functions. Neural networks training algorithms. Vanilla gradient descent and stochastic gradient descent Deep learning. Convolutional neural networks. Structure of the convolutional neural network. The purpose of the convolutional block Unsupervised learning. Cluster analysis. Comparison of clustering approaches. Clustering problem statement. Clustering score. Methods for calculating the distance between clusters Clustering with graphs. Clustering with minimum spanning forest. Prim's algorithm. Kruskal's algorithm The k-means method. Problem statement and computational steps (with an example illustration). k-means initialization. The k-means++ algorithm. Choosing the number of clusters. The elbow method Hierarchical agglomerative clustering. Dendrogram. Types of metrics in a feature

	space. Methods for calculating the distance between clusters. Ward's method
4. Замятин А.В. Интеллектуальный анализ данных: учеб. пособие. – Томск: Издательский Дом государственного университета, 2020. – 196 с. URL:https://www.litres.ru/a-v-zamyatin/intellektualnyy-analiz-dannyh-67267130/ (not free)	Regression analysis. Binary data classification problem. Linear regression model. Multiple linear regression. Ordinary least squares method. Calculating linear regression coefficients. L1 and L2 regularization Metric space. Euclidean space. Taxicab geometry. The k-nearest neighbors algorithm. The weighted k-nearest neighbors algorithm Decision tree. Decision rules. The Gini impurity. Information gain. Classification and regression trees. Ensemble learning for classification and regression. Bagging. Boosting Regression scoring. Classification scores and margins. Performance of a binary classifier. The confusion matrix. Confusion-matrix-based performance measures. Analysis of the coefficient of determination and the adjusted coefficient of determination. The ROC-curve: calculation, properties, and interpretation. The ROC-curve of a perfect and random classifiers. ROC AUC. The ROC AUC calculation method The mathematical model of an artificial neuron. Activation functions. Feed-forward neural networks. The mathematical model of a multi-layer neural network. Neural network training as an optimization problem. Backpropagation. Calculating the gradient of the objective function. Loss function. Types of loss functions. Comparison of the learning processes with different loss functions. Neural networks training algorithms. Vanilla gradient descent and stochastic gradient descent Unsupervised learning. Cluster analysis. Comparison of clustering approaches. Clustering problem statement. Clustering score. Methods for calculating the distance between clusters The k-means method. Problem statement and computational steps (with an example illustration). k-means initialization. The k-means++ algorithm. Choosing the number of clusters. The elbow method

	Hierarchical agglomerative clustering. Dendrogram. Types of metrics in a feature space. Methods for calculating the distance between clusters. Ward's method Dimensionality reduction. Principal component analysis. Independent component analysis
5. Хайкин С. Нейронные сети: Полный курс. – М.: Издательский дом «Вильямс», 2006. URL:https://vk.com/doc10903696_282205957?hash=f65JFiOVscYSbo89bNB46X2zkuLo129srM6yZN2FMUg&dl=k4ljqZRojL4eoVU6I20gLQYXG7wOzOGKcgvqQ3OZWkD (free) URL:https://www.labirint.ru/books/529153/ (not free)	The k-means method. Problem statement and computational steps (with an example illustration). k-means initialization. The k-means++ algorithm. Choosing the number of clusters. The elbow method Hierarchical agglomerative clustering. Dendrogram. Types of metrics in a feature space. Methods for calculating the distance between clusters. Ward's method Dimensionality reduction. Principal component analysis. Independent component analysis

Sources in English	**Corresponding topic**
1. Bishop C. M. Pattern recognition and machine learning. – springer, 2006. URL:http://users.isr.ist.utl.pt/~wurmd/Livros/school/Bishop%20-%20Pattern%20Recognition%20And%20Machine%20Learning%20-%20Springer%20%202006.pdf (free)	Regression analysis. Binary data classification problem. Linear regression model. Multiple linear regression. Ordinary least squares method. Calculating linear regression coefficients. L1 and L2 regularization Metric space. Euclidean space. Taxicab geometry. The k-nearest neighbors algorithm. The weighted k-nearest neighbors algorithm Bayes' theorem. Bayesian decision rule. Naive Bayes classifier Decision tree. Decision rules. The Gini impurity. Information gain. Classification and regression trees. Ensemble learning for classification and regression. Bagging. Boosting Regression scoring. Classification scores and margins. Performance of a binary classifier. The confusion matrix. Confusion-matrix-based performance measures. Analysis of the coefficient of determination and the adjusted coefficient of determination. The ROC-curve: calculation, properties, and interpretation. The ROC-curve of a perfect and random classifiers.

	ROC AUC. The ROC AUC calculation method The mathematical model of an artificial neuron. Activation functions. Feed-forward neural networks. The mathematical model of a multi-layer neural network. Neural network training as an optimization problem. Backpropagation. Calculating the gradient of the objective function. Loss function. Types of loss functions. Comparison of the learning processes with different loss functions. Neural networks training algorithms. Vanilla gradient descent and stochastic gradient descent Deep learning. Convolutional neural networks. Structure of the convolutional neural network. The purpose of the convolutional block Unsupervised learning. Cluster analysis. Comparison of clustering approaches. Clustering problem statement. Clustering score. Methods for calculating the distance between clusters Clustering with graphs. Clustering with minimum spanning forest. Prim's algorithm. Kruskal's algorithm The k-means method. Problem statement and computational steps (with an example illustration). k-means initialization. The k-means++ algorithm. Choosing the number of clusters. The elbow method Hierarchical agglomerative clustering. Dendrogram. Types of metrics in a feature space. Methods for calculating the distance between clusters. Ward's method
2. Murphy, K. P. (2022). Probabilistic Machine Learning: An introduction. MIT Press. URL:https://github.com/probml/pml-book/releases/latest/download/book1.pdf	Regression analysis. Binary data classification problem. Linear regression model. Multiple linear regression. Ordinary least squares method. Calculating linear regression coefficients. L1 and L2 regularization Metric space. Euclidean space. Taxicab geometry. The k-nearest neighbors algorithm. The weighted k-nearest neighbors algorithm Bayes' theorem. Bayesian decision rule. Naive Bayes classifier Decision tree. Decision rules. The Gini impurity. Information gain. Classification and regression trees. Ensemble learning for

	classification and regression. Bagging. Boosting Regression scoring. Classification scores and margins. Performance of a binary classifier. The confusion matrix. Confusion-matrix-based performance measures. Analysis of the coefficient of determination and the adjusted coefficient of determination. The ROC-curve: calculation, properties, and interpretation. The ROC-curve of a perfect and random classifiers. ROC AUC. The ROC AUC calculation method The mathematical model of an artificial neuron. Activation functions. Feed-forward neural networks. The mathematical model of a multi-layer neural network. Neural network training as an optimization problem. Backpropagation. Calculating the gradient of the objective function. Loss function. Types of loss functions. Comparison of the learning processes with different loss functions. Neural networks training algorithms. Vanilla gradient descent and stochastic gradient descent Deep learning. Convolutional neural networks. Structure of the convolutional neural network. The purpose of the convolutional block Unsupervised learning. Cluster analysis. Comparison of clustering approaches. Clustering problem statement. Clustering score. Methods for calculating the distance between clusters Clustering with graphs. Clustering with minimum spanning forest. Prim's algorithm. Kruskal's algorithm The k-means method. Problem statement and computational steps (with an example illustration). k-means initialization. The k-means++ algorithm. Choosing the number of clusters. The elbow method Hierarchical agglomerative clustering. Dendrogram. Types of metrics in a feature space. Methods for calculating the distance between clusters. Ward's method Dimensionality reduction. Principal component analysis. Independent component analysis

3. Friedman J. et al. The elements of statistical learning. – New York: Springer series in statistics, 2001. – T. 1. – №. 10. URL:https://hastie.su.domains/Papers/ESLII.pdf (free)	Regression analysis. Binary data classification problem. Linear regression model. Multiple linear regression. Ordinary least squares method. Calculating linear regression coefficients. L1 and L2 regularization Metric space. Euclidean space. Taxicab geometry. The k-nearest neighbors algorithm. The weighted k-nearest neighbors algorithm Bayes' theorem. Bayesian decision rule. Naive Bayes classifier Decision tree. Decision rules. The Gini impurity. Information gain. Classification and regression trees. Ensemble learning for classification and regression. Bagging. Boosting Regression scoring. Classification scores and margins. Performance of a binary classifier. The confusion matrix. Confusion-matrix-based performance measures. Analysis of the coefficient of determination and the adjusted coefficient of determination. The ROC-curve: calculation, properties, and interpretation. The ROC-curve of a perfect and random classifiers. ROC AUC. The ROC AUC calculation method The mathematical model of an artificial neuron. Activation functions. Feed-forward neural networks. The mathematical model of a multi-layer neural network. Neural network training as an optimization problem. Backpropagation. Calculating the gradient of the objective function. Loss function. Types of loss functions. Comparison of the learning processes with different loss functions. Neural networks training algorithms. Vanilla gradient descent and stochastic gradient descent Deep learning. Convolutional neural networks. Structure of the convolutional neural network. The purpose of the convolutional block Unsupervised learning. Cluster analysis. Comparison of clustering approaches. Clustering problem statement. Clustering score. Methods for calculating the distance between clusters Clustering with graphs. Clustering with minimum spanning forest. Prim's algorithm. Kruskal's algorithm

	The k-means method. Problem statement and computational steps (with an example illustration). k-means initialization. The k-means++ algorithm. Choosing the number of clusters. The elbow method Hierarchical agglomerative clustering. Dendrogram. Types of metrics in a feature space. Methods for calculating the distance between clusters. Ward's method
4. Goodfellow I. et al. Deep learning. – Cambridge: MIT press, 2016. URL:http://imlab.postech.ac.kr/dkim/class/csed514_2019s/DeepLearningBook.pdf (free)	Regression analysis. Binary data classification problem. Linear regression model. Multiple linear regression. Ordinary least squares method. Calculating linear regression coefficients. L1 and L2 regularization Metric space. Euclidean space. Taxicab geometry. The k-nearest neighbors algorithm. The weighted k-nearest neighbors algorithm Bayes' theorem. Bayesian decision rule. Naive Bayes classifier Decision tree. Decision rules. The Gini impurity. Information gain. Classification and regression trees. Ensemble learning for classification and regression. Bagging. Boosting Regression scoring. Classification scores and margins. Performance of a binary classifier. The confusion matrix. Confusion-matrix-based performance measures. Analysis of the coefficient of determination and the adjusted coefficient of determination. The ROC-curve: calculation, properties, and interpretation. The ROC-curve of a perfect and random classifiers. ROC AUC. The ROC AUC calculation method The mathematical model of an artificial neuron. Activation functions. Feed-forward neural networks. The mathematical model of a multi-layer neural network. Neural network training as an optimization problem. Backpropagation. Calculating the gradient of the objective function. Loss function. Types of loss functions. Comparison of the learning processes with different loss functions. Neural networks training algorithms. Vanilla gradient descent and stochastic gradient descent The k-means method. Problem statement and computational steps (with an example

	illustration). k-means initialization. The k-means++ algorithm. Choosing the number of clusters. The elbow method Hierarchical agglomerative clustering. Dendrogram. Types of metrics in a feature space. Methods for calculating the distance between clusters. Ward's method Dimensionality reduction. Principal component analysis. Independent component analysis
5. Murphy K. P. Machine learning: a probabilistic perspective. – MIT press, 2012. URL:http://noiselab.ucsd.edu/ECE228/Murphy_Machine_Learning.pdf (free)	Regression analysis. Binary data classification problem. Linear regression model. Multiple linear regression. Ordinary least squares method. Calculating linear regression coefficients. L1 and L2 regularization Metric space. Euclidean space. Taxicab geometry. The k-nearest neighbors algorithm. The weighted k-nearest neighbors algorithm Bayes' theorem. Bayesian decision rule. Naive Bayes classifier Decision tree. Decision rules. The Gini impurity. Information gain. Classification and regression trees. Ensemble learning for classification and regression. Bagging. Boosting Regression scoring. Classification scores and margins. Performance of a binary classifier. The confusion matrix. Confusion-matrix-based performance measures. Analysis of the coefficient of determination and the adjusted coefficient of determination. The ROC-curve: calculation, properties, and interpretation. The ROC-curve of a perfect and random classifiers. ROC AUC. The ROC AUC calculation method The mathematical model of an artificial neuron. Activation functions. Feed-forward neural networks. The mathematical model of a multi-layer neural network. Neural network training as an optimization problem. Backpropagation. Calculating the gradient of the objective function. Loss function. Types of loss functions. Comparison of the learning processes with different loss functions. Neural networks training algorithms. Vanilla gradient descent and stochastic gradient descent

	Deep learning. Convolutional neural networks. Structure of the convolutional neural network. The purpose of the convolutional block Unsupervised learning. Cluster analysis. Comparison of clustering approaches. Clustering problem statement. Clustering score. Methods for calculating the distance between clusters Clustering with graphs. Clustering with minimum spanning forest. Prim's algorithm. Kruskal's algorithm The k-means method. Problem statement and computational steps (with an example illustration). k-means initialization. The k-means++ algorithm. Choosing the number of clusters. The elbow method Hierarchical agglomerative clustering. Dendrogram. Types of metrics in a feature space. Methods for calculating the distance between clusters. Ward's method Dimensionality reduction. Principal component analysis. Independent component analysis
6. Simon H. Neural networks: a comprehensive foundation. – Prentice hall, 1999. URL:https://www.amazon.com/Neural-Networks-Comprehensive-Foundation-2nd/dp/0132733501 (not free)	Regression analysis. Binary data classification problem. Linear regression model. Multiple linear regression. Ordinary least squares method. Calculating linear regression coefficients. L1 and L2 regularization Metric space. Euclidean space. Taxicab geometry. The k-nearest neighbors algorithm. The weighted k-nearest neighbors algorithm Bayes' theorem. Bayesian decision rule. Naive Bayes classifier Decision tree. Decision rules. The Gini impurity. Information gain. Classification and regression trees. Ensemble learning for classification and regression. Bagging. Boosting Regression scoring. Classification scores and margins. Performance of a binary classifier. The confusion matrix. Confusion-matrix-based performance measures. Analysis of the coefficient of determination and the adjusted coefficient of determination. The ROC-curve: calculation, properties, and interpretation. The ROC-curve of a perfect and random classifiers.

	ROC AUC. The ROC AUC calculation method The mathematical model of an artificial neuron. Activation functions. Feed-forward neural networks. The mathematical model of a multi-layer neural network. Neural network training as an optimization problem. Backpropagation. Calculating the gradient of the objective function. Loss function. Types of loss functions. Comparison of the learning processes with different loss functions. Neural networks training algorithms. Vanilla gradient descent and stochastic gradient descent Unsupervised learning. Cluster analysis. Comparison of clustering approaches. Clustering problem statement. Clustering score. Methods for calculating the distance between clusters Clustering with graphs. Clustering with minimum spanning forest. Prim's algorithm. Kruskal's algorithm The k-means method. Problem statement and computational steps (with an example illustration). k-means initialization. The k-means++ algorithm. Choosing the number of clusters. The elbow method Hierarchical agglomerative clustering. Dendrogram. Types of metrics in a feature space. Methods for calculating the distance between clusters. Ward's method Dimensionality reduction. Principal component analysis. Independent component analysis

Section 6. Discrete mathematics

Sources in Russian	Corresponding topic
1. Дж. Андерсон. Дискретная математика и комбинаторика Москва-Санкт-Петербург, 2017 URL:https://studizba.com/files/show/djvu/2898-1-dzh-anderson--diskretnaya-matematika-i.html (free)	Division with a remainder. GCD and LCM. The Euclidean algorithm. Diophantine equations. Prime numbers. Factorization methods Positional notation of natural numbers. Algorithms for converting numbers between bases

	Arithmetic of remainders. Linear comparisons. Chinese remainder theorem. System of residual classes Euler's totient function. Fermat's little theorem. Euler's theorem. RSA encryption. General formulas of combinatorics. Enumerative combinatorics. The inclusion-exclusion principle Graph. Graph properties. Depth-first search and breadth-first search. Connectivity of graphs. Kosaraju's algorithm. Planar graph. Euler's theorem An Eulerian trail in a graph. A Hamiltonian path in a graph. Dijkstra's algorithm. The Floyd-Warshall algorithm Spanning trees in a graph. Prim's algorithm. Kruskal's algorithm
2. Н. Кристофидес Теория графов: алгоритмический подход, М.: Мир, 1997 URL:https://studizba.com/files/show/pdf/53991-1-n-kristofides--teoriya-grafov.html (free)	Graph. Graph properties. Depth-first search and breadth-first search. Connectivity of graphs. Kosaraju's algorithm. Planar graph. Euler's theorem An Eulerian trail in a graph. A Hamiltonian path in a graph. Dijkstra's algorithm. The Floyd-Warshall algorithm Spanning trees in a graph. Prim's algorithm. Kruskal's algorithm
3. Харари Теория графов, urss 2018, изд.5, дополненное URL:https://stugum.files.wordpress.com/2014/03/harary-graph-theory.pdf (free)	Graph. Graph properties. Depth-first search and breadth-first search. Connectivity of graphs. Kosaraju's algorithm. Planar graph. Euler's theorem An Eulerian trail in a graph. A Hamiltonian path in a graph. Dijkstra's algorithm. The Floyd-Warshall algorithm Spanning trees in a graph. Prim's algorithm. Kruskal's algorithm

Sources in English	**Corresponding topic**
1. F. Harary. Graph Theory. – CRC Press. 1969. URL:https://www.taylorfrancis.com/books/mono/10.1201/9780429493768/graph-theory-frank-harary (not free)	Graph. Graph properties. Depth-first search and breadth-first search. Connectivity of graphs. Kosaraju's algorithm. Planar graph. Euler's theorem An Eulerian trail in a graph. A Hamiltonian path in a graph. Dijkstra's algorithm. The Floyd-Warshall algorithm

	Spanning trees in a graph. Prim's algorithm. Kruskal's algorithm
2. Hopcroft, Motvani, Ullman. Introduction to automata theory, languages and computations, 2001. URL:https://www-2.dc.uba.ar/staff/becher/Hopcroft-Motwani-Ullman-2001.pdf (free)	Boolean functions. Normal forms of Boolean functions. Duality of functions. Zhegalkin polynomials. Closed classes. Post's theorem Logic of statements. Predicate logic. Resolution method Formal languages and grammars. Context-free grammars. Automatic grammars. Finite automata. Determinacy. Kleene's theorem Turing machine. Markov algorithms. Partially recursive functions
3. J. Anderson. Discrete Mathematics With Combinatorics. – Prentice Hall, 2003 URL:https://www.amazon.com/Discrete-Mathematics-Combinatorics-James-Anderson/dp/0130457914 (not free)	Division with a remainder. GCD and LCM. The Euclidean algorithm. Diophantine equations. Prime numbers. Factorization methods Positional notation of natural numbers. Algorithms for converting numbers between bases Arithmetic of remainders. Linear comparisons. Chinese remainder theorem. System of residual classes Euler's totient function. Fermat's little theorem. Euler's theorem. RSA encryption. General formulas of combinatorics. Enumerative combinatorics. The inclusion-exclusion principle
4. N. Christofides. Graph Theory. An algorithmic Approach. Academic Press, 1975. URL:https://www.amazon.com/Graph-Theory-Algorithmic-Approach-Christofides/dp/0121743500 (not free)	Graph. Graph properties. Depth-first search and breadth-first search. Connectivity of graphs. Kosaraju's algorithm. Planar graph. Euler's theorem An Eulerian trail in a graph. A Hamiltonian path in a graph. Dijkstra's algorithm. The Floyd-Warshall algorithm Spanning trees in a graph. Prim's algorithm. Kruskal's algorithm
5. P. Linz. An Introduction to Formal Languages and Automata, Jones & Barlett Learning, 2011. URL:http://www.its.caltech.edu/~matilde/FormalLanguageTheory.pdf (free) URL:https://www.amazon.com/Introduction-Formal-Languages-Automata-5th/dp/144961552X (not free)	Boolean functions. Normal forms of Boolean functions. Duality of functions. Zhegalkin polynomials. Closed classes. Post's theorem Logic of statements. Predicate logic. Resolution method Formal languages and grammars. Context-free grammars. Automatic grammars. Finite automata. Determinacy. Kleene's theorem Turing machine. Markov algorithms. Partially recursive functions

6. T. Cormen, C. Leiserson, R. Rivest, C. Stein. Introduction to algorithms. – MIT Press, 2009. URL:https://sd.blackball.lv/library/Introduction_to_Algorithms_Third_Edition_(2009).pdf (free)	Graph. Graph properties. Depth-first search and breadth-first search. Connectivity of graphs. Kosaraju's algorithm. Planar graph. Euler's theorem An Eulerian trail in a graph. A Hamiltonian path in a graph. Dijkstra's algorithm. The Floyd-Warshall algorithm Spanning trees in a graph. Prim's algorithm. Kruskal's algorithm

Recommended online courses

Section 1. Linear algebra and analytic geometry

1. Matrix Algebra for Engineers. (Coursera)

URL: https://www.coursera.org/learn/matrix-algebra-engineers

2. Linear Algebra - Foundations to Frontiers. (edX)

URL: https://www.edx.org/course/linear-algebra-foundations-to-frontiers

3. Precalculus. Unit: Matrices. (Khan Academy)

URL: https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:matrices

4. First Steps in Linear Algebra for Machine Learning. (Coursera)

URL: https://www.coursera.org/learn/first-steps-in-linear-algebra-for-machine-learning

5. Matrix Methods. (Coursera)

URL: https://www.coursera.org/learn/matrix-methods

6. GTx's Introductory Linear Algebra. (edX)

URL: https://www.edx.org/professional-certificate/gtx-introductory-linear-algebra

7. Precalculus. Unit: Vectors. (Khan Academy)

URL: https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:vectors

8. Precalculus. Unit: Conic sections. (Khan Academy)

URL: https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:conics

9. Mathematics 1. Unit: Analytic geometry. (Khan Academy)

URL: https://www.khanacademy.org/math/math1/x89d82521517266d4:analytic-geo

10. Three Dimensional Geometry. (toppr)

URL: https://www.toppr.com/guides/maths/three-dimensional-geometry/

11. Pre-University Calculus. (edX)

URL: https://www.edx.org/course/pre-university-calculus-2

Section 2. Real and complex analysis (calculus)

1. Differential Calculus through Data and Modeling. (Coursera)

URL: https://www.coursera.org/specializations/differential-calculus-data-modeling#courses

2. Introduction to Calculus. (Coursera)

URL: https://www.coursera.org/learn/introduction-to-calculus

3. Calculus: Single Variable Part 1 – Functions. (Coursera)

URL: https://www.coursera.org/learn/single-variable-calculus

4. Calculus: Single Variable Part 2 – Differentiation. (Coursera)

URL: https://www.coursera.org/learn/differentiation-calculus

5. Calculus: Single Variable Part 3 – Integration. (Coursera)
URL: https://www.coursera.org/learn/integration-calculus
6. Calculus: Single Variable Part 4 – Applications. (Coursera)
URL: https://www.coursera.org/learn/applications-calculus
7. Complex Analysis (edX)
URL: https://www.edx.org/course/complex-analysis
8. Introduction to Complex Analysis (Coursera)
URL: https://www.coursera.org/learn/complex-analysis
9. Fundamentals of Complex Variables Analysis. (Udemy)
URL: https://www.udemy.com/course/fundamentals-of-complex-variables-analysis/
10. Complex Variables and Transforms. (Udemy)
URL: https://www.udemy.com/course/complex-variables-and-transforms/
11. Complex Analysis. (Nptel)
URL: https://nptel.ac.in/courses/111/103/111103070/#

Section 3. Differential equations

1. Integral Calculus. Unit: Differential equations. (Khan Academy)
URL: https://www.khanacademy.org/math/integral-calculus/ic-diff-eq
2. Math. Unit: Differential equations. (Khan Academy)
URL: https://www.khanacademy.org/math/differential-equations
3. Ordinary Differential Equations and Linear Algebra - Part 1. (edX)
URL: https://www.edx.org/course/ordinary-differential-equations-and-linear-algebra
4. Ordinary Differential Equations. (Udemy)
URL: https://www.udemy.com/course/ordinary-differential-equations/
5. Differential Equations for Engineers. (Coursera)
URL: https://www.coursera.org/learn/differential-equations-engineers
6. Introduction to Ordinary Differential Equations. (Coursera)
URL: https://www.coursera.org/learn/ordinary-differential-equations

Section 4. Theory of probability and mathematical statistics

1. Probability Theory, Statistics and Exploratory Data Analysis. (Coursera)
URL: https://www.coursera.org/learn/probability-theory-statistics
2. Probability Theory: Foundation for Data Science. (Coursera)
URL: https://www.coursera.org/learn/probability-theory-foundation-for-data-science
3. Introduction to Statistics. (Coursera)
URL: https://www.coursera.org/learn/stanford-statistics
4. Combinatorics and Probability. (Coursera)
URL: https://www.coursera.org/learn/combinatorics
5. Introduction to Statistics: Probability. (edX)
URL: https://www.edx.org/course/introduction-to-statistics-probability-2
6. Probability and Statistics I: A Gentle Introduction to Probability (edX)
URL:https://www.edx.org/course/probability-and-statistics-i-a-gentle-introduction-to-probability
7. Probability Theory (Stepik)
URL: https://stepik.org/52134

Section 5. Machine learning

1. Machine Learning (Coursera)
URL: https://www.coursera.org/learn/machine-learning
2. Deep Learning (Coursera)
URL: https://www.coursera.org/specializations/deep-learning
3. Advanced Machine Learning (Coursera)
URL: https://www.coursera.org/specializations/aml
4. IBM Machine Learning (Coursera)
URL: https://www.coursera.org/professional-certificates/ibm-machine-learning
5. Machine Learning (Coursera)
URL: https://www.coursera.org/specializations/machine-learning
6. Mathematics for Machine Learning (Coursera)
URL: https://www.coursera.org/specializations/mathematics-machine-learning
7. Machine Learning for All (Coursera)
URL: https://www.coursera.org/learn/uol-machine-learning-for-all

Section 6. Discrete mathematics

1. Introduction to graph theory (class central)
URL: https://www.classcentral.com/course/graphs-9213
2. Discrete mathematics (class central)
URL: https://www.classcentral.com/course/discrete-mathematics-8133
3. Graph Theory (Coursera)
URL: https://www.coursera.org/learn/graphs
4. Mathematics for Computer Science (Coursera)
URL:https://www.coursera.org/learn/mathematics-for-computer-science?page=2&index=prod_all_launched_products_term_optimization