

Program: Applied Mathematics & Artificial Intelligence

This document outlines the scope of themes, which may be included in the Olympiad tests. The themes are grouped by areas and are followed by a list of recommended literature in the Russian and English languages.

Olympiad winner's skill set

You should be familiar with:

1. data analysis methods based on the principles of fundamental mathematics;
2. the fundamentals of how artificial intelligence works;
3. the basic concepts of modern mathematics.

You should be able to:

1. choose and justify methods for solving assigned tasks;
2. use modern software for data processing;
3. apply knowledge from across various branches of mathematics to obtain known solutions to research problems.

You should possess skills in:

1. creating statistical reports, graphs, charts and diagrams, basic models for selected data;
2. predicting the results of research activities (basic skills in formulating scientific hypotheses);
3. proving well-known modern mathematical theorems (within your specialization).

Content

Section 1. Linear algebra and analytic geometry

1. Polynomials. Degree of polynomial, root of polynomial, multiplicity of a root. Little Bezout's theorem. Polynomial division with a remainder. Greatest common divisor of polynomials. Interpolation polynomial in Lagrange and Newton forms.
2. Complex numbers. Algebraic and trigonometric forms. Computation of a root of the complex number. Main theorem of algebra.
3. Systems of linear algebraic equations. Cramer's rule. Rouche–Capelli theorem. General solution of a system of linear equations
4. Vector spaces. Basis, dimension of a vector space. Subspaces. Sum and intersection of vector subspaces. Linear transformation of a finite-dimensional vector space, the matrix of a linear transformation. Changing the matrix of a linear transformation with the change of basis
5. Eigenvectors and eigenvalues, their properties
6. Euclidean spaces. Linear transformations on Euclidean spaces
7. Bilinear and quadratic forms
8. Vectors and their properties. Dot (scalar), cross (vector) and triple scalar (mixed) products
9. Ways to define a line and a plane. Calculating the angle between a plane and a line, and between two lines. Calculating the distance from a point to a line and the distance between two lines
10. Second-order curves (conic sections). Ellipse, parabola, hyperbola
11. Second-order surfaces (quadric surfaces). Ellipsoid, one-sheeted hyperboloid, two-sheeted hyperboloid, elliptic paraboloid, hyperbolic paraboloid, cone

12. Affine transformations: definition and properties
13. Orthogonal transformations: definition and properties

Section 2. Real and complex analysis (calculus)

1. Limit of a sequence: definition and properties. Cauchy's criterion. Limit inferior and limit superior. The Bolzano–Weierstrass theorem. The limit of a function at a point. The equivalence of Cauchy's and Heine's definitions
2. Function continuity at a point. Behavior of a continuous function on a segment: the Weierstrass and Bolzano-Cauchy theorems. The inverse function theorem. Uniform continuity, Cantor's theorem.
3. Derivative of a function (of a single variable) at a point: definition and basic properties. The derivative of a composition of functions. Differentiability of a function at a point. Function differential at a point. Derivative of an inverse function. Higher order derivatives and differentials. The Leibniz rule. Rolle's Theorem, the mean value theorem, Cauchy's mean value theorem. L'Hôpital's rule. Taylor's expansion with the Peano and Lagrange form of the remainder. Using Taylor's expansion and L'Hôpital's rule for limit calculations. Using derivative to study the properties of a function of a single variable: monotonicity, extrema, convexity, inflection points
4. Indefinite integral. Definite integral. Darboux's criterion of the integrability of a function. Properties of an integral with a variable upper limit: continuity, differentiability. The Newton–Leibniz formula. Geometric applications of definite integral. Improper integrals. Absolute convergence and conditional convergence. Cauchy's criterion, Dirichlet's test of convergence
5. Differentiability of a function of several variables. Necessary conditions and sufficient conditions for differentiability. Gradient of a function. Implicit function theorem. Local extrema of a function of several variables. Necessary conditions and sufficient conditions of a local extremum point. Conditional extrema of a function. The Lagrange multiplier method, necessary and sufficient conditions for conditional extrema
6. Numeric series. Absolute and conditional convergence. Cauchy's criterion, comparison test, integral test, the Root and Ratio tests, Leibniz's and Dirichlet's tests. Function series. Uniform convergence. Cauchy's criterion, the Weierstrass test, Dirichlet's test of uniform convergence. Power series. The radius of convergence, the Cauchy—Hadamard formula. The Taylor expansion. Taylor expansions for elementary functions
7. Line integral. Green's theorem. Surface integrals. The Gauss—Ostrogradsky theorem. Stokes' theorem
8. A Fourier series. Pointwise convergence conditions. Uniform convergence conditions. The Fourier transform of an absolutely integrable function and its properties. The Fourier transform of a derivative and the derivative of a Fourier transform
9. Complex numbers: definition, properties
10. Analytic functions
11. Integral of a function of a complex variable. Cauchy's integral theorem. Cauchy's integral formula
12. Maximum modulus principle and the Schwarz lemma
13. The Taylor and Laurent series
14. Residues of complex functions. Cauchy's residue theorem. Calculation of integrals using residues. Jordan's lemma
15. Conformal mappings. Möbius transformations. The Joukowski transform

Section 3. Differential equations

1. Ordinary differential equations. Separation of variables. Reduction of order of the differential equation
2. Linear ODEs and systems of ODEs with constant coefficients
3. Linear ODEs and systems of ODEs with variable coefficients. A fundamental solution set. The Wrońskian, Liouville's formula. The variation of constants method
4. Equilibrium of an autonomous system of differential equations. Classifying the equilibria of linear autonomous second-order systems. Stability and asymptotic stability of equilibrium. First integrals of autonomous system of differential equations. Theorem on the number of independent first integrals
5. Sturm-Liouville problem. Energy identity and weak solution. Green function. Spectre of Sturm-Liouville operator.
6. Linear PDEs. General solution and the Cauchy problem
7. Calculus of variations. The Euler–Lagrange equation. Necessary condition for a weak local extremum

Section 4. Theory of probability and mathematical statistics

1. Fundamentals of combinatorics: counting rules, addition and multiplication rules, combinations with and without repetition, binomial coefficients and Newton's binomial theorem
2. Probability space. Independent events. Summation theorem. Conditional probability. Collectively exhaustive events. Law of total probability. Bayes' formula
3. A random variable and its cumulative distribution function. Expectation and variance of a random variable: definition and properties
4. Basic classes of probability distributions: binomial, geometric, uniform, Poisson, exponential, normal. Bernoulli trials. Chebyshev's inequality. Law of large numbers
5. Joint probability distributions. Independence. Covariance. Correlation coefficient
6. Estimating the parameters of a distribution. Statistical hypothesis testing

Section 5. Machine learning

1. Regression analysis. Binary data classification problem. Linear regression model. Multiple linear regression. Ordinary least squares method. Calculating linear regression coefficients. L1 and L2 regularization
2. Metric space. Euclidean space. Taxicab geometry. The k-nearest neighbors algorithm. The weighted k-nearest neighbors algorithm
3. Bayes' theorem. Bayesian decision rule. Naive Bayes classifier
4. Decision tree. Decision rules. The Gini impurity. Information gain. Classification and regression trees. Ensemble learning for classification and regression. Bagging. Boosting
5. Regression scoring. Classification scores and margins. Performance of a binary classifier. The confusion matrix. Confusion-matrix-based performance measures. Analysis of the coefficient of determination and the adjusted coefficient of determination. The ROC-curve: calculation, properties, and interpretation. The ROC-curve of a perfect and random classifiers. ROC AUC. The ROC AUC calculation method
6. The mathematical model of an artificial neuron. Activation functions. Feed-forward neural networks. The mathematical model of a multi-layer neural network. Neural network training as an optimization problem. Backpropagation. Calculating the gradient of the objective function. Loss function. Types of loss functions. Comparison of the learning processes with different loss functions. Neural networks training algorithms. Vanilla gradient descent and stochastic gradient descent

7. Deep learning. Convolutional neural networks. Structure of the convolutional neural network. The purpose of the convolutional block
8. Unsupervised learning. Cluster analysis. Comparison of clustering approaches. Clustering problem statement. Clustering score. Methods for calculating the distance between clusters
9. Clustering with graphs. Clustering with minimum spanning forest. Prim's algorithm. Kruskal's algorithm
10. The k-means method. Problem statement and computational steps (with an example illustration). k-means initialization. The k-means++ algorithm. Choosing the number of clusters. The elbow method
11. Hierarchical agglomerative clustering. Dendrogram. Types of metrics in a feature space. Methods for calculating the distance between clusters. Ward's method
12. Dimensionality reduction. Principal component analysis. Independent component analysis

Section 6. Discrete Mathematics

1. Division with a remainder. GCD and LCM. The Euclidean algorithm. Diophantine equations. Prime numbers. Factorization methods
2. Positional notation of natural numbers. Algorithms for converting numbers between bases
3. Arithmetic of remainders. Linear comparisons. Chinese remainder theorem. System of residual classes
4. Euler's totient function. Fermat's little theorem. Euler's theorem. RSA encryption.
5. General formulas of combinatorics. Enumerative combinatorics. The inclusion-exclusion principle
6. Graph. Graph properties. Depth-first search and breadth-first search. Connectivity of graphs. Kosaraju's algorithm. Planar graph. Euler's theorem
7. Eulerian trail in a graph. A Hamiltonian path in a graph. Dijkstra's algorithm. The Floyd-Warshall algorithm
8. Spanning trees in a graph. Prim's algorithm. Kruskal's algorithm
9. Boolean functions. Normal forms of Boolean functions. Duality of functions. Zhegalkin polynomials. Closed classes. Post's theorem
10. Logic of statements. Predicate logic. Resolution method
11. Formal languages and grammars. Context-free grammars. Automatic grammars. Finite automata. Determinacy. Kleene's theorem
12. Turing machine. Markov algorithms. Partially recursive functions

RECOMMENDED LITERATURE

Section 1. Linear algebra and analytic geometry

Sources in Russian	Topic
1. В.А. Ильин, Э.Г. Позняк. Аналитическая геометрия, любое издание, напр., М.: Физматлит, 2004. URL: https://www.labirint.ru/books/585885/ (not free)	Topics 6-11
2. В.А. Ильин, Э.Г. Позняк. Линейная алгебра, любое издание, напр., М.: ФИЗМАТЛИТ, 2007. URL: https://obuchalka.org/2017092496602/	Topics 1-5

lineinaya-algebra-ilin-v-a-poznyak-e-g-2005.html (free)	
<p>3. Д. В. Беклемишев. Курс аналитической геометрии и линейной алгебры, любое издание, напр., М.: Физматлит, 2005. URL: http://mathdep.ifmo.ru/wp-content/uploads/2020/09/%D0%9A%D1%83%D1%80%D1%81-%D0%B0%D0%BD%D0%B0%D0%BB%D0%B8%D1%82%D0%B8%D1%87%D0%B5%D1%81%D0%BA%D0%BE%D0%B9-%D0%B3%D0%B5%D0%BE%D0%BC%D0%B5%D1%82%D1%80%D0%B8%D0%B8-%D0%B8-%D0%BB%D0%B8%D0%BD%D0%B5%D0%B9%D0%BD%D0%BE%D0%B9-%D0%B0%D0%BB%D0%B3%D0%B5%D0%B1%D1%80%D1%8B.-%D0%91%D0%B5%D0%BA%D0%BB%D0%B5%D0%BC%D0%B8%D1%88%D0%B5%D0%B2-%D0%94.%D0%92..pdf (free)</p>	All topics in section 1
<p>4. И.И. Привалов. Аналитическая геометрия. СПб.: Лань, 2010. URL: https://www.litres.ru/ivan-privalov/analiticheskaya-geometriya-40-e-izd-uchebnik-dlya-v-62697081/ (not free)</p>	Topics 6-11
<p>5. И.М. Гельфанд. Лекции по линейной алгебре, любое издание, напр., М.: Добросвет, МЦНМО, 1998. URL: https://www.phantastike.com/math/lek_po_lineynoy_algebre/pdf/ (free)</p>	Topics 1-5
<p>6. П.С. Александров. Лекции по аналитической геометрии, любое издание, напр., СПб.: Лань, 2016. URL: https://www.ozon.ru/product/lektcii-po-analiticheskoy-geometrii-popolnennye-neobhodimymi-svedeniyami-iz-algebry-aleksandrov-320039875/?sh=dK3WCN9s_g (not free)</p>	Topics 6-11
<p>7. Э.Б. Винберг. Курс алгебры, любое издание, напр., М.: МЦНМО, 2011. URL: http://mathprofi.com/uploads/files/2581_f41_e.b.vinberg-kurs-algebry-2-e-izd.pdf?key=d04a1718e76a1b8366c8fc0d4d87caf3 (free)</p>	Topics 1-5

Sources in English	Topic
<p>1. A. C. Burdette. Analytic Geometry. Academic Press, 1971. URL:https://www.amazon.com/Analytic-Geometry-C-Burdette-ebook/dp/B01DUEBGW8 (not free) URL:https://www.amazon.com/Introduction-Analytic-Geometry-Calculus/dp/B00201CRGW (not free)</p>	Topics 6-11
<p>2. B. Spain. Analytical Geometry. Pergamon, 1963. URL:https://download.tuxfamily.org/openmathdep/geometry_analytic/Analytical_Geometry-Spain.pdf (free)</p>	Topics 6-11
<p>3. I. M. Gel'fand. Lectures on Linear Algebra. Dover Publications, 1989. URL:https://www.amazon.com/Lectures-Linear-Algebra-Dover-Mathematics/dp/0486660826 (not free)</p>	Topics 1-5
<p>4. R. Bronson, J.T. Saccoman, G. Costa. Linear Algebra: introduction. Academic Press, 2013. URL:https://mathematicalolympiads.files.wordpress.com/2012/08/linear-algebra2.pdf (free)</p>	Topics 1-5
<p>5. S. Axler. Linear Algebra Done Right. Springer, 2015. URL:https://www.amazon.com/Linear-Algebra-Right-Undergraduate-Mathematics/dp/0387982582 (not free) URL:http://alefenu.com/libri/LADR.pdf (free)</p>	Topics 1-5
<p>6. S. Andrilli, D.Hecker. Elementary Linear Algebra. Academic Press, 2016. URL:https://www.amazon.com/Elementary-Linear-Algebra-Stephen-Andrilli/dp/0123747511 (not free)</p>	Topics 1-5
<p>7. V. A. Ilyin, E. G. Poznyak . Linear Algebra. Collets, 1986. URL:https://urss.ru/cgi-bin/db.pl (not free) https://www.amazon.com/Linear-Algebra-V-Ilyin/dp/0828533407 (not free)</p>	Topics 1-5
<p>8. W.R. Gondin, B. Sohmer. Intermediate Algebra & Analytic Geometry. Made Simple, 1965. URL:https://www.amazon.com/Intermediate-Algebra-Analytic-Geometry-</p>	All topics in section 1

William/dp/1483256707 (not free)	
9. Ya. S. Bugrov, S.M. Nikolsky. Fundamentals of Linear Algebra and Analytical Geometry. Mir, 1982. URL: https://vdocuments.net/mir-bugrov-y-s-and-nikolsky-s-m-fundamentals-of-linear-algebra-and.html?page=3 (free)	All topics in section 1

Section 2. Real and complex analysis (calculus)

Sources in Russian	Topic
1. А.Г. Свешников, А. Н. Тихонов. Теория функции комплексного переменного, любое издание, напр., М. Физматлит, 2005. URL: http://read.newlibrary.ru/read.php/pdf=15234 (free)	Topics 9-15
2. А. И. Маркушевич. Краткий курс теории аналитических функций, любое издание, напр., М.: Мир, 2006. URL: https://obuchalka.org/20200827124103/kratkii-kurs-teorii-analiticheskikh-funkcii-markushevich-a-i.html (free)	Topics 9-15
3. И. И. Привалов. Введение в теорию функции комплексного переменного, любое издание, напр., М.: Наука, 1984. URL: http://elibrary.sgu.ru/uch_lit/559.pdf (free)	Topics 9-15
4. С.А. Теляковский. Курс лекций по математическому анализу, семестры 1, 2, 3. М.: МИАН, 2009, 2011, 2013. URL: http://www.mathnet.ru/links/c94dc3095bc0709f9821e3986416a9d8/lkn11.pdf (сем. 1) (free) URL: http://www.mathnet.ru/links/b07162e094c98d486a21ca0069e0cf08/lkn17.pdf (сем. 2) (free) URL: http://www.mathnet.ru/links/094a6c684e3207db229087e38dab9dbf/lkn20.pdf (сем. 3) (free)	Topics 1-8
5. С. М. Никольский. Курс математического анализа. В 2-х томах. М.: Физматлит, 2001. URL: http://www.physics.gov.az/book_K/NIKOLSKI2.PDF (free)	Topics 1-8
6. Фукс Б.А., Шабат Б.В. Функции комплексного переменного и некоторые их приложения. М: Наука, 1964.	Topics 9-15

URL: https://ikfia.ysn.ru/wp-content/uploads/2018/01/FuksShabat1964ru.pdf (free)	
7. Ю. В. Сидоров, М. В. Федорюк, М. И. Шабунин. Лекции по теории функций комплексного переменного, любое издание, напр., М.: Наука, 1982. URL: http://mph1.phys.spbu.ru/~badanin/3kypc/FilesAdd/Shabunin_TFKP.pdf (free)	Topics 9-15

Sources in English	Topic
1. A.G. Sveshnikov, A.N. Tikhonov. The Theory of Functions of a Complex Variable. Mir, 1978. URL: https://download.tuxfamily.org/openmathdep/analysis_complex/Functions_Complex_Variable-Sveshnikov.pdf (free)	Topics 9-15
2. D.E. Marshall. Complex Analysis. Cambridge University Press, 2019. URL: https://www.matem.unam.mx/~hector/[Lars_Ahlfors]_Complex_Analysis_(Third_Edition).pdf (free) URL: https://www.amazon.com/Complex-Analysis-Cambridge-Mathematical-Textbooks/dp/110713482X (not free)	Topics 9-15
3. G. Strang. Calculus. 3 rd edition. Wellesley-Cambridge Press, 2017. URL: https://ocw.mit.edu/ans7870/resources/Strang/Edited/Calculus/Calculus.pdf (free) URL: https://www.amazon.com/Calculus-3rd-Gilbert-Strang/dp/0980232759 (not free)	Topics 1-8
4. H. Flanders, R.R. Korfhage, J.J. Price. A Second Course in Calculus. Academic Press, 1974. URL: https://www.amazon.com/Second-Course-Calculus-Harley-Flanders-ebook/dp/B01DUEGPE2 (not free)	Topics 1-8
5. L. Ahlfors, L. V., Complex analysis. McGraw-Hill, 1986. URL: https://www.matem.unam.mx/~hector/[Lars_Ahlfors]_Complex_Analysis_(Third_Edition).pdf (free) URL: https://www.amazon.com/Complex-Analysis-Lars-Ahlfors/dp/0070006571 (not free)	Topics 9-15
6. L. Loomis, S. Steinberg. Advanced	Topics 1-8

Calculus. Revised edition, Landom-Boston, 1990 URL: https://people.math.harvard.edu/~shlomo/docs/Advanced_Calculus.pdf (free)	
7. M. I. Shabunin, Yu. V. Sidorov, M. V. Fedoryuk. Lectures on the Theory of Functions of a Complex Variable. Mir, 1985. URL: https://www.amazon.com/Lectures-Functions-Variable-I-Shabunin-Facsimile/dp/B07L35NPHT (not free) URL: http://theory.fi.infn.it/colomo/metodi/esti/Sidorov_Fedoryuk_Shabunin.pdf (free)	Topics 9-15
8. Reiner Kuhnau. Handbook of Complex Analysis. North Holland, 2004. URL: https://www.amazon.com/Handbook-Complex-Analysis-Reiner-Kuhnau/dp/0444828451 (not free)	Topics 9-15
9. T. Dence, J. Dence. Advanced Calculus. Academic Press, 2009. URL: https://www.amazon.com/Advanced-Calculus-Transition-Thomas-Dence/dp/0123749557 (not free)	Topics 1-8
10. Ya. S. Bugrov, S. M. Nikolsky. Differential and Integral Calculus. Imported Pubn, 1983. URL: https://www.amazon.com/Differential-Integral-Calculus-Ya-Bugrov/dp/0828523061 (not free)	Topics 1-8

Section 3. Differential equations

Sources in Russian	Topic
1. А.Н. Тихонов, А.Б. Васильева, А.Г. Свешников. Дифференциальные уравнения: Учеб.: Для вузов. — 4-е изд. — М.: ФИЗМАТЛИТ, 2005. URL: https://obuchalka.org/2015031483302/differencialnie-uravneniya-tihonov-a-n-vasileva-a-b-sveshnikov-a-g-2005.html (free) URL: https://vk.com/doc409016625_601016842?hash=tbYzqszejwezTgmyNzyEC9At2fdvwFmBiORxNbGZxRMO&dl=KPO5Udumkyy1NaN2qzuuNdcW1IROCOF1ZHKLWFNErT (free)	Topics 1-4
2. А.П. Карташев, Б.Л. Рождественский. Обыкновенные дифференциальные	Topics 1-4,6

<p>уравнения и основы вариационного исчисления. М.: Наука, 1980. URL:https://obuchalka.org/2013040270515/obiknovennie-differencialnie-uravneniya-i-osnovi-variacionnogo-ischisleniya-kartashev-a-p-rojdestvenskii-b-l-1980.html (free)</p>	
<p>3. А.Ф. Филиппов. Введение в теорию дифференциальных уравнений. М.: КомКнига, 2007. URL:https://obuchalka.org/2014032376465/vvedenie-v-teoriu-differencialnih-uravnenii-filippov-a-f-2007.html (free)</p>	Topics 1-4
<p>4. А.Ф. Филиппов. Сборник задач по дифференциальным уравнениям. Ижевск: НИЦ "Регулярная и хаотическая динамика", 2000. URL:http://kvm.gubkin.ru/pub/uok/FilippovDU.pdf (free)</p>	Topics 1-4
<p>5. Л.Э. Эльсгольц. Дифференциальные уравнения и вариационное исчисление. М.: Наука, 1965. URL:http://www.phys.nsu.ru/balakina/EI%27sgol%27dz_Dif_ur_i_var_isch.pdf (free)</p>	Topics 1-4,6
<p>6. М.В. Федорюк. Обыкновенные дифференциальные уравнения. М.: Наука, 1985. URL:http://cmcestuff.esyr.org/vmkbotva-r15/2%20%D0%BA%D1%83%D1%80%D1%81/4%20%D0%A1%D0%B5%D0%BC%D0%B5%D1%81%D1%82%D1%80/%D0%94%D0%B8%D1%84%D1%84%D1%83%D1%80%D1%8B/%D0%A3%D1%87%D0%B5%D0%B1%D0%BD%D0%B8%D0%BA%D0%B8/%D0%A4%D0%B5%D0%B4%D0%BE%D1%80%D1%8E%D0%BA%20%D0%9C.,%20%D0%9E%D0%B1%D1%8B%D0%BA%D0%BD%D0%BE%D0%B2%D0%B5%D0%BD%D0%BD%D1%8B%D0%B5%20%D0%B4%D0%B8%D1%84%D1%84.%D1%83%D1%80%D0%B0%D0%B2%D0%BD%D0%B5%D0%BD%D0%B8%D1%8F,%201985.pdf (free)</p>	Topics 1-4
<p>7. Н.М. Матвеев. Методы интегрирования обыкновенных дифференциальных уравнений. Учебник. Минск: «Вышэйшая школа», 1974. URL:https://obuchalka.org/2015033083660/metodi-integrirvaniya-obiknovennih-</p>	Topics 1-4

differencialnih-uravnenii-matveev-n-m-1967.html (free) URL: https://cdnpdf.com/pdf-12290-metody-integrirovaniya-obyknovennyh-differencialnyh-uravnenij-matveev-nm (free)	
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Sources in English	Topic
1. G.F. Carrier, C.E. Pearson. Partial Differential Equations. Academic Press, 1976. URL: https://www.amazon.com/Partial-Differential-Equations-Theory-Technique-dp-0121604500/dp/0121604500/ref=mt_other?encoding=UTF8&me=&qid= (not free)	Topic 5
2. G. Simmons. Differential equations with applications and historical notes. McGraw-Hill, New York, 1991. URL: https://caisatech.net/uploads/120220%20Differential%20equations.pdf (free)	Topics 1-4
3. G. Strang. Differential equations and Linear algebra. Wellesley-Cambridge Press, 2014. URL: https://www.amazon.com/Differential-Equations-Linear-Algebra-Gilbert/dp/0980232791#:~:text=Differential%20equations%20and%20linear%20algebra%20are%20two%20central%20topics%20in,giving%20increased%20flexibility%20to%20instructors (not free) URL: https://math.mit.edu/~gs/dela/ (free)	Topics 1-4
4. L. Elsgolts. Differential Equations and the Calculus of Variations. University Press of the Pacific, 2003. URL: https://ia800908.us.archive.org/2/items/ElsgoltsDifferentialEquationsAndTheCalculusOfVariations/Elsgolts-Differential-Equations-and-the-Calculus-of-Variations.pdf (free) URL: https://www.amazon.com/Differential-Equations-Calculus-Variations-Elsgolts/dp/1410210677 (not free)	Topics 1-4,6
5. M. Tenenbaum, H. Pollard. Ordinary Differential Equations. Dover Publications, 1985. URL: https://netsanet4all.files.wordpress.com/2018/12/294222977-ordinary-	Topics 1-4

[differential-equations-tenenbaum-pollard-0486649407.pdf](https://obuchalka.org/20190716111540/differential-equations-tenenbaum-pollard-0486649407.pdf) (free)

Section 4. Theory of probability and mathematical statistics

Sources in Russian	Topic
<p>1. Б.В. Гнеденко. Курс теории вероятностей. 8-е изд., испр. и доп.—М.: Едиториал УРСС, 2005. URL:https://obuchalka.org/20190716111540/kurs-teorii-veroyatnostei-uchebnik-gnedenko-b-v-2011.html (free)</p>	Topics 1-5
<p>2. В. П. Чистяков. Курс теории вероятностей, любое издание URL:https://obuchalka.org/20210219129486/kurs-teorii-veroyatnostei-chistyakov-v-p-2000.html (free) URL:https://vk.com/doc409016625_585522734?hash=shU94s8Mkzaz5HeKr1ywtzg9SrhLDNtiQACwqblc5zk&dl=56IpiYMTeuz3eGbp6mbOGSR1yNcyUBZ80zEA05ykrbL (free)</p>	Topics 1-5
<p>3. В. Феллер. Введение в теорию вероятностей и ее приложения, любое издание. URL:https://www.labyrinth.ru/books/828835/ (not free) URL:https://vk.com/doc409016625_541696520?hash=WCRmDd8Eiwo6wglbHb3UEgzAKoxswEkGwwRizPtgqKc&dl=OWNlrVPbnWq1foZAAKUq944tkgs20FoTsDXkU9LETI4 (free) URL:https://vk.com/doc409016625_541697907?hash=09ukiWbbZdvhqbsNPXPFZHdAO0TUJSWF3qxvTMHJ5S4&dl=NwYIWvIFGAE8zJ7Qe9vTZ98gXzwDnaaWwtzh5ZdKAwX (free)</p>	Topics 1-5
<p>4. Г.И. Ивченко, Ю.И. Медведев. Математическая статистика. М.: Высш. шк., 1984. URL:https://www.hse.ru/pubs/share/direct/content_document/103185710 (free)</p>	Topic 6
<p>5. Е. С. Вентцель. Теория вероятностей, 2006 URL:https://obuchalka.org/20190227107251/teoriya-veroyatnostei-ventcel-e-s-2006.html (free)</p>	Topics 1-5

Sources in English	Topic
1. B.V. Gnedenko. Theory of Probability. CRC Press, 1998. URL: https://www.amazon.com/Theory-Probability-Boris-V-Gnedenko/dp/9056995855 (not free)	Topics 1-5
2. E.L. Lehmann, G. Casella. Theory of Point Estimation (2nd ed.). New York: Springer, 1998. URL: https://www.dcehvpm.org/E-Content/Stat/E%20L%20Lehaman.pdf (free)	Topic 6
3. E.L. Lehmann, Joseph P. Romano. Testing Statistical Hypotheses (3rd ed.). New York: Springer, 2005. URL: https://sites.stat.washington.edu/jaw/COURSES/580s/582/HO/Lehmann and Romano-TestingStatisticalHypotheses.pdf (free)	Topic 6
4. G.R. Grimmett, D.R. Stirzaker. Probability and Random Processes. Oxford University Press, 2001. URL: http://old-eclass.uop.gr/modules/document/file.php/TST244/%5BGeoffrey R. Grimmett%20David R. Stirzaker%5D Probabi%28BookZ.org%29.pdf (free)	Topics 1-5
5. R.E. Walpol, R.H. Myers, S.L. Myers, K. Ye. Probability and Statistics. Prentice Hall, 2011. URL: https://spada.uns.ac.id/pluginfile.php/221008/mod_resource/content/1/ProbabilityStatistics for EngineersScientists%289th Edition%29 Walpole.pdf (free)	All topics in section 4
6. V.K. Zakharov, V. P. Chistyakov, B. A. Sevastyanov. Probability Theory for Engineers. Optimization Software. 1987. URL: https://www.alibris.com/booksearch/detail?invid=17109690527&isbn=9780911575132&utm_medium=affiliate&utm_source=GuWPtmTDDdQ&utm_campaign=2&siteID=GuWPtmTDDdQ-bX4XKw.oyIGLVgYhZ53VdQ (not free)	Topics 1-5
7. W. Feller. An Introduction to Probability Theory and its Applications. John Wiley & Sons, 1967. URL: https://www.amazon.com/Introduction-Probability-Theory-Applications-Vol/dp/0471257087 (not free)	Topics 1-5

URL: https://bitcoinwords.github.io/assets/papers/an-introduction-to-probability-theory-and-its-applications.pdf (free)	
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Section 5. Machine learning

Sources in Russian	Topic
1. Архангельская Е. Глубокое обучение / С. Николенко, А. Кагурин. – СПб: Издательский дом "Питер", 2017. URL: https://www.ozon.ru/product/glubokoe-obuchenie-arhangelskaya-e-o-kadurin-a-a-211432536/?sh=dK3WCCOfsA (not free)	All topics in section 5
2. Гудфеллоу Я., Иошуа Б., Курвилль А. Глубокое обучение. – Litres, 2018. URL: https://www.ozon.ru/product/glubokoe-obuchenie-tsvetnye-illyustratsii-bendzhio-ioshua-gudfellou-yan-217046706/?sh=dK3WCMY0oA (not free)	All topics in section 5
3. Воронцов К. В. Математические методы обучения по прецедентам (теория обучения машин) // Москва. – 2011. – С. 141. URL: http://www.machinelearning.ru/wiki/images/6/6d/voron-ml-1.pdf (free)	All topics in section 5
4. Замятин А.В. Интеллектуальный анализ данных: учеб. пособие. – Томск: Издательский Дом государственного университета, 2020. – 196 с. URL: https://www.litres.ru/a-v-zamyatin/intellektualnyy-analiz-dannyh-67267130/ (not free)	All topics in section 5
5. Хайкин С. Нейронные сети: Полный курс. – М.: Издательский дом «Вильямс», 2006. URL: http://i.uran.ru/webcab/system/files/bo_oxpdf/neyronnye-seti-polnyy-kurs/229022.pdf (free) URL: https://www.labirint.ru/books/529153/ (not free)	All topics in section 5

Sources in English	Topic
1. Bishop C. M. Pattern recognition and machine learning. – springer, 2006. URL: http://users.isr.ist.utl.pt/~wurmd/Livros/school/Bishop%20-%20Pattern%20Recognition%20And%20M	All topics in section 5

achine%20Learning%20-%20Springer%20%202006.pdf (free)	
2. Brownlee J. Statistical methods for machine learning. Discover how to transform data into knowledge with Python. Machine Learning Mastery. – 2020. URL: https://machinelearningmastery.com/statistics-for-machine-learning/ (not free) URL: https://vk.com/doc270219984_558991959?hash=RIVSLsu6DP8ee45SNvzXh7azCp61PJcCe7VaQOEUzTk&dl=BGqIqfLcSRB4LG7wrZalbE2ICu2s3xTaEWAR3fg0zKL (free)	All topics in section 5
3. Friedman J. et al. The elements of statistical learning. – New York: Springer series in statistics, 2001. – Т. 1. – №. 10. URL: https://hastie.su.domains/Papers/ESLII.pdf (free)	All topics in section 5
4. Goodfellow I. et al. Deep learning. – Cambridge: MIT press, 2016. URL: http://imlab.postech.ac.kr/dkim/class/used514_2019s/DeepLearningBook.pdf (free)	All topics in section 5
5. Murphy K. P. Machine learning: a probabilistic perspective. – MIT press, 2012. URL: http://noiselab.ucsd.edu/ECE228/Murphy_Machine_Learning.pdf (free)	All topics in section 5
6. Simon H. Neural networks: a comprehensive foundation. – Prentice hall, 1999. URL: https://www.amazon.com/Neural-Networks-Comprehensive-Foundation-2nd/dp/0132733501 (not free)	All topics in section 5

Section 6. Discrete mathematics

Sources in Russian	Topic
1. Дж. Андерсон. Дискретная математика и комбинаторика Москва-Санкт-Петербург, 2017 URL: https://studizba.com/files/show/djvu/2898-1-dzh-anderson--diskretnaya-matematika-i.html (free)	Topics 1-8
2. Н. Кристофидес Теория графов: алгоритмический подход, М.: Мир, 1997	Topics 6-8

URL: https://studizba.com/files/show/pdf/53991-1-n-kristofides--teoriya-grafov.html (free)	
3. Харари Теория графов, urss 2018, изд.5, дополненное URL: https://stugum.files.wordpress.com/2014/03/harary-graph-theory.pdf (free)	Topics 6-8

Sources in English	Topic
1. F. Harary. Graph Theory. – CRC Press. 1969. URL: https://www.taylorfrancis.com/books/mono/10.1201/9780429493768/graph-theory-frank-harary (not free)	Topics 6-8
2. Hopcroft, Motvani, Ullman. Introduction to automata theory, languages and computations, 2001. URL: https://e.famnit.upr.si/pluginfile.php/636821/mod_page/content/8/Automata.pdf (free)	Topics 9-12
3. J. Anderson. Discrete Mathematics With Combinatorics. – Prentice Hall, 2003 URL: https://www.amazon.com/Discrete-Mathematics-Combinatorics-James-Anderson/dp/0130457914 (not free)	Topics 1-5
4. N. Christofides. Graph Theory. An algorithmic Approach. Academic Press, 1975. URL: https://www.amazon.com/Graph-Theory-Algorithmic-Approach-Christofides/dp/0121743500 (not free)	Topics 6-8
5. P. Linz. An Introduction to Formal Languages and Automata, Jones & Barlett Learning, 2011. URL: http://www.its.caltech.edu/~matilde/FormalLanguageTheory.pdf (free) URL: https://www.amazon.com/Introduction-Formal-Languages-Automata-5th/dp/144961552X (not free)	Topics 9-12
6. T. Cormen, C. Leiserson, R. Rivest, C. Stein. Introduction to algorithms. – MIT Press, 2009. URL: https://sd.blackball.lv/library/Introduction to Algorithms Third Edition (2009).pdf (free)	Topics 6-8

RECOMMENDED ONLINE COURSES

Section 1. Linear algebra and analytic geometry

1. Matrix Algebra for Engineers. (Coursera)
URL: <https://www.coursera.org/learn/matrix-algebra-engineers>
2. Linear Algebra - Foundations to Frontiers. (edX)
URL: <https://www.edx.org/course/linear-algebra-foundations-to-frontiers>
3. Precalculus. Unit: Matrices. (Khan Academy)
URL: <https://www.khanacademy.org/math/prec calculus/x9e81a4f98389efdf:matrices>
4. Linear Algebra for Machine Learning and Data Science (Coursera)
<https://www.coursera.org/learn/machine-learning-linear-algebra>
5. Matrix Methods. (Coursera)
URL: <https://www.coursera.org/learn/matrix-methods>
6. GTX's Introductory Linear Algebra. (edX)
URL: <https://www.edx.org/professional-certificate/gtx-introductory-linear-algebra>
7. Precalculus. Unit: Vectors. (Khan Academy)
URL: <https://www.khanacademy.org/math/prec calculus/x9e81a4f98389efdf:vectors>
8. Precalculus. Unit: Conic sections. (Khan Academy)
URL: <https://www.khanacademy.org/math/prec calculus/x9e81a4f98389efdf:conics>
9. Mathematics 1. Unit: Analytic geometry. (Khan Academy)
URL: <https://www.khanacademy.org/math/math1/x89d82521517266d4:analytic-geo>
10. Three Dimensional Geometry. (toppr)
URL: <https://www.toppr.com/guides/maths/three-dimensional-geometry/>
11. Pre-University Calculus. (edX)
URL: <https://www.edx.org/course/pre-university-calculus-2>

Section 2. Real and complex analysis (calculus)

1. Differential Calculus through Data and Modeling. (Coursera)
URL: <https://www.coursera.org/specializations/differential-calculus-data-modeling#courses>
2. Introduction to Calculus. (Coursera)
URL: <https://www.coursera.org/learn/introduction-to-calculus>
3. Calculus: Single Variable Part 1 – Functions. (Coursera)
URL: <https://www.coursera.org/learn/single-variable-calculus>
4. Calculus: Single Variable Part 2 – Differentiation. (Coursera)
URL: <https://www.coursera.org/learn/differentiation-calculus>
5. Calculus: Single Variable Part 3 – Integration. (Coursera)
URL: <https://www.coursera.org/learn/integration-calculus>
6. Calculus: Single Variable Part 4 – Applications. (Coursera)
URL: <https://www.coursera.org/learn/applications-calculus>
7. Complex Analysis (edX)
URL: <https://www.edx.org/course/complex-analysis>
8. Introduction to Complex Analysis (Coursera)
URL: <https://www.coursera.org/learn/complex-analysis>
9. Fundamentals of Complex Variables Analysis. (Udemy)
URL: <https://www.udemy.com/course/fundamentals-of-complex-variables-analysis/>
10. Complex Variables and Transforms. (Udemy)
URL: <https://www.udemy.com/course/complex-variables-and-transforms/>
11. Complex Analysis. (Nptel)
URL: <https://nptel.ac.in/courses/111/103/111103070/#>

Section 3. Differential equations

1. Integral Calculus. Unit: Differential equations. (Khan Academy)
URL: <https://www.khanacademy.org/math/integral-calculus/ic-diff-eq>
2. Math. Unit: Differential equations. (Khan Academy)
URL: <https://www.khanacademy.org/math/differential-equations>
3. Ordinary Differential Equations and Linear Algebra - Part 1. (edX)
URL: <https://www.edx.org/course/ordinary-differential-equations-and-linear-algebra>
4. Ordinary Differential Equations. (Udemy)
URL: <https://www.udemy.com/course/ordinary-differential-equations/>
5. Differential Equations for Engineers. (Coursera)
URL: <https://www.coursera.org/learn/differential-equations-engineers>
6. Introduction to Ordinary Differential Equations. (Coursera)
URL: <https://www.coursera.org/learn/ordinary-differential-equations>

Section 4. Theory of probability and mathematical statistics

1. Probability Theory, Statistics and Exploratory Data Analysis. (Coursera)
URL: <https://www.coursera.org/learn/probability-theory-statistics>
2. Probability Theory: Foundation for Data Science. (Coursera)
URL: <https://www.coursera.org/learn/probability-theory-foundation-for-data-science>
3. Introduction to Statistics. (Coursera)
URL: <https://www.coursera.org/learn/stanford-statistics>
4. Probability & Statistics for Machine Learning & Data Science (Coursera)
<https://www.coursera.org/learn/machine-learning-probability-and-statistics>
5. Introduction to Statistics (Coursera)
<https://www.coursera.org/learn/stanford-statistics>
6. Probability and Statistics: To p or not to p? (Coursera)
<https://www.coursera.org/learn/probability-statistics>
7. Probability Theory (Stepik)
URL: <https://stepik.org/52134>

Section 5. Machine learning

1. Machine Learning (Coursera)
URL: <https://www.coursera.org/learn/machine-learning>
2. Deep Learning (Coursera)
URL: <https://www.coursera.org/specializations/deep-learning>
3. Advanced Machine Learning (Coursera)
URL: <https://www.coursera.org/specializations/aml>
4. IBM Machine Learning (Coursera)
URL: <https://www.coursera.org/professional-certificates/ibm-machine-learning>
5. Machine Learning (Coursera)
URL: <https://www.coursera.org/specializations/machine-learning>
6. Mathematics for Machine Learning (Coursera)
URL: <https://www.coursera.org/specializations/mathematics-machine-learning>
7. Machine Learning for All (Coursera)
URL: <https://www.coursera.org/learn/uol-machine-learning-for-all>

Section 6. Discrete mathematics

1. Introduction to graph theory (class central)

PROGRAM

URL: <https://www.classcentral.com/course/graphs-9213>

2. Discrete mathematics (class central)

URL: <https://www.classcentral.com/course/discrete-mathematics-8133>

3. Graph Theory (Coursera)

URL: <https://www.coursera.org/learn/graphs>

4. Mathematics for Computer Science (Coursera)

URL: https://www.coursera.org/learn/mathematics-for-computer-science?page=2&index=prod_all_launched_products_term_optimization