Physical Sciences: second-round sample tasks

Module 1. Test questions with one correct answer (2 points for the correct answer)

1.1 A fly crawls one quarter of the circle at a constant speed v in time t. The distance covered by the fly is:

a. vt

b. $2vt/\pi$

c. $2\sqrt{2}vt/\pi$

d. 0

1.2 Where on the surface of the Earth is the gravitational acceleration modulus greatest?

a. at latitude 45°

b. at the latitude of Cairo

c. at the equator

d. at the poles

1.3 How does the root mean square velocity of ideal gas particles change if the temperature of the gas is quadrupled?

a. increases 4 times

b. decreases 4 times

c. does not change

d. increases by 2 times

1.4 How does the density of an ideal gas change if its temperature and pressure are doubled simultaneously?

a. increases 4 times

b. decreases 4 times

c. does not change

d. halves

1.5 An insulating sphere is uniformly charged over its volume. The center of the sphere is located at some distance from a uniformly charged infinite insulating plane thin sheet. This distance is bigger than the radius of the sphere. How will the force of interaction between the sphere and the plane change if this distance is doubled?

a. The force will double

b. The force will halve

c. The force will decrease 4 times

d. The force will not change

1.6 How will the inductance of a coil change if its length is doubled?

a. The inductance will double

b. The inductance will halve

c. The inductance will quadruple.

d. The inductance will decrease 4 times

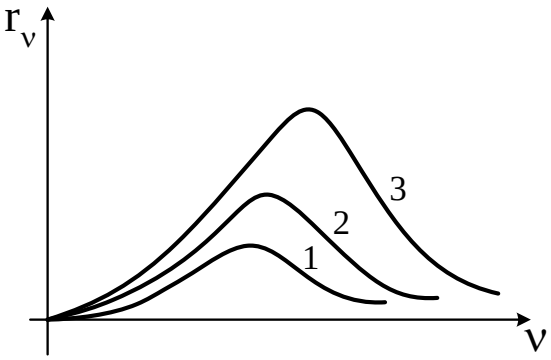
1.7 A beam of natural light is incident on a flat boundary between two media with an angle of incidence of 55°. The reflected beam is linearly polarized. What is the angle of refraction?

a. 15°
b. 25°
c. 35°
d. 45°

1.8 An object is located at a distance of 7 cm from a thin converging lens on its optical axis. The focal length of the lens is 5 cm. What is its linear magnification?

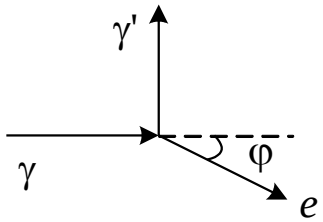
a. 1.5
b. 2
c. 2.5
d. 3

1.9 The figure shows the dependence of the spectral emittance r_ν for radiation of a blackbody on the frequency. Specify which of the curves shown on the graph corresponds to the highest temperature.



a. 1
b. 2
c. 3
d. All curves correspond to the same temperature

1.10 In the Compton effect, photon scattering on an electron is observed. The figure shows the directions of the incident photon (γ), the scattered photon (γ') and the recoiling electron (*e*). The angle between the directions of the scattered photon and the incident photon is 90°, the angle between the direction of the recoiling electron motion and the incident photon is $\varphi = 30°$. What is the scattered photon momentum, if the momentum of the incident photon is P_γ?



a. $P_\gamma / \sqrt{3}$
b. 0.5 P_γ

c. $\sqrt{3}$ P□
d. $1.5\sqrt{3}$ P□

Module 2. Tasks with a numerical answer that is compared with the reference one (7 points for the right answer)

2.1 Find the water pressure force of the reservoir with a depth of h = 50 m on a dam with a length of L = 400 m. The density of water is ρ = 1000 kg/m^3, the free fall acceleration is g = 10 N/kg. Give the answer in Giga Newton.
Answer: 5

2.2 Suppose η_0 is the ratio of the molecular concentration of hydrogen to that of nitrogen at the Earth's surface, while η is the corresponding ratio at the height h = 1500 m. Find the ratio η/η_0 at the temperature T = 300 K, assuming that the temperature and the free fall acceleration $g = 9.8\ \text{m/s}^2$ are independent of the height. The molar masses of hydrogen and nitrogen are M_1 = 2 g/mole and M_2 = 28 g/mole, respectively. The universal gas constant $R = 8.314\ \text{J/(K}\cdot\text{mole)}$. Round your answer to the second decimal figure.
Answer: 1.17

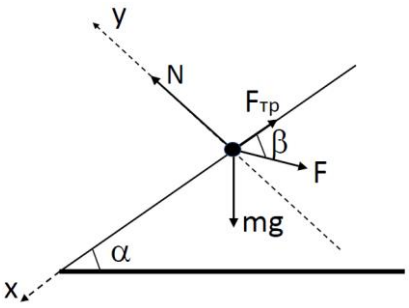
2.3 A point electric dipole is located at some distance from a point charge and is oriented along a straight line connecting the dipole and the charge. How many times in absolute value will the force acting on the dipole decrease if it is rotated by 90 degrees?
Answer: 2

2.4 The distance between the grooves of a diffraction grating is d = 4 μm. Light with wavelength λ = 0.55 μm is incident normally on the grating. What is the highest order of diffraction maximum that can be observed?
Answer: 7

2.5 An electron in a hydrogen atom is in a state with the principal quantum number n=2. Find the number of different states of the electron corresponding to the given n, taking into account the spin of the electron.
Answer: 8

Module 3. Tasks with a detailed answer (The maximum score for a correct and reasoned solution to each problem is 15 points. Points are also awarded for a partial solution to the problem, depending on its contribution to the complete solution)

3.1. A sledge of the mass $m = 10$ kg is placed on an ice slide with a slope angle $\alpha = 30^\circ$. The coefficient of friction between the sledge and the surface μ = 0.1. What is the minimum force required to keep the sledge in equilibrium? The free fall acceleration $g = 9.8\ m/s^2$. Give the answer in kN and round it to the second decimal figure.
Solution:



Since $\mu < tg\alpha$, the sledge cannot be in mechanical equilibrium. To keep the sledge in equilibrium, force $\mathbf{F}$ must be applied at some angle β to the tilted plane to prevent the sledge from sliding.

The balance of forces providing equilibrium is

$$\mathbf{F} + m\mathbf{g} + \mathbf{N} + \mathbf{F}_{mp} = 0, \qquad (3.1.1)$$

where $\mathbf{F}_{mp}$ is the maximal force of the static friction, $\mathbf{N}$ is the normal force of the reaction.

(5 points)

For x- and y-axis projections of the equation (3.1.1), we have

$$mg\sin\alpha - F\cos\beta - \mu N = 0, \qquad (3.1.2)$$

and

$$N - mg\cos\alpha - F\sin\beta = 0, \qquad (3.1.3)$$

respectively. Here we take into account that $F_{mp} = \mu N$.

Based on the equations (3.1.2) and (3.1.3), we find

$$F = \frac{\sin\alpha - \mu\cos\alpha}{\cos\beta + \mu\sin\beta} mg. \qquad (3.1.4)$$

(5 points)

According to expression (3.1.4), force F is minimum, when the denominator $\varphi(\beta) = \cos\beta + \mu\sin\beta$ is maximum. The function $\varphi(\beta)$ has the maximum value if

$$tg\beta = \mu. \qquad (3.1.5)$$

(2 points)

Consequently, the minimal value of the force is

$$F = \frac{\sin\alpha - \mu\cos\alpha}{\sqrt{1+\mu^2}} mg \qquad (3.1.6)$$

(1 point)

Hence, we find $F \approx 0.04$ кN

(2 points)

3.2 An ideal gas is expanded isothermally from a volume of 0.1 m^3 to a volume of 0.3 m^3. The final gas pressure is $2\cdot10^5$ Pa. Determine the amount of heat transferred to the gas during this process. Give the answer in kJ and round it to the integer value.

Solution: According to the first law of thermodynamics, the amount of heat imparted to the gas during a process is

$$Q = \Delta U + W. \qquad (3.2.1)$$

Since the process is isothermal, the increment of the internal energy $\Delta U = 0$, and the problem is reduced to calculating the work done by the gas when expanding from volume V_1=0.1 m^3 to volume V_2=0.3 m^3

$$Q = W\,. \qquad (3.2.2)$$

(3 points)

This work is

$$W = \int_{V_1}^{V_2} p dV\,, \qquad (3.2.3)$$

where p is the pressure of the gas as a function of its volume.

(2 points)

This function $p(V)$ is given by the equation of an ideal gas state

$$p = \frac{\nu RT}{V}\,. \qquad (3.2.4)$$

(2 point)

Substituting (3.2.4) into (3.2.3) and taking into account that the amount of substance ν and the temperature T in the gas remain constant during the process, we obtain

$$W = \nu RT \int_{V_1}^{V_2} \frac{dV}{V} = \nu RT \ln\left(\frac{V_2}{V_1}\right). \qquad (3.2.5)$$

(3 points)

Expressing the unknown νRT from the gas state equation in terms of the final pressure $p_2 = 2\cdot 10^5$ Pa and the final volume V_2=0.3 m^3, we get

$$W = p_2 V_2 \ln\left(\frac{V_2}{V_1}\right). \qquad (3.2.6)$$

(3 points)

Hence,

$W \approx 66$ kJ (3.2.7)

(2 points)

3.3 Two identical empty parallel-plate capacitors are half filled with the same dielectric. However, the first one was filled in such a way that the vacuum-dielectric interface was perpendicular to the capacitor plates, and in the second the vacuum-dielectric interface was parallel to the plates. The capacitance of the first capacitor is 25/16 times greater than the capacitance of the second capacitor. What is the permittivity ε of the dielectric? Edge effects should be neglected.

Solution: Let C be the capacitance of an empty capacitor. The first capacitor can be considered as two capacitors connected in parallel, the capacitance of which is equal to C/2 and εC/2 respectively **(3 points)**.

Then the capacitance of the first capacitor is $C_1 = (\varepsilon + 1)C/2$ **(3 points)**.

The second capacitor can be represented as 2 capacitors connected in series, the capacitance of which is equal to 2C and 2εC **(3 points)**.

Then the capacitance of the second capacitor can be calculated as $1/C_2 = 1/(2C)+1/(2\varepsilon C)$ **(3 points)**. Multiplying these 2 equations, we get

$25/16=(1+\varepsilon)^2/(4\varepsilon)$.

Hence, $\varepsilon = 4$ **(3 points)**.