

## Physical sciences & Technology: second-round sample tasks

**Module 1. Test questions with one correct answer (worth 1 point). Correct answers are given in red.**

### Mechanics

**1.1 A fly has crawled one-quarter of the circle at a constant speed  $v$  over time  $t$ . The distance covered by the fly is:**

- A)  $vt$
- B)  $2vt/\pi$
- C)  $2\sqrt{2}vt/\pi$
- D) 0

**1.2 Where on the surface of Earth is the gravitational acceleration modulus the greatest?**

- A) at latitude  $45^\circ$
- B) at the latitude of Cairo
- C) at the equator
- D) at the poles

**1.3 What is the moment of inertia of a thin solid disk of mass  $m$  and radius  $R$  relative to an axis within its plane and positioned at a distance equal to one-quarter of the radius from its center?**

- A)  $3mR^2/16$
- B)  $mR^2/2$
- C)  $mR^2/4$
- D)  $5mR^2/16$

**1.4 A particle of mass  $m$  moves along a circle of radius  $R$  with uniform tangential acceleration  $a_\tau$ . Find the angular momentum of the particle relative to the center of the circle as a function of time  $t$ .**

- A)  $ma_\tau t 2\pi R$
- B)  $ma_\tau t R$
- C)  $mR^2/t$
- D)  $m\sqrt{a_\tau} R^3$

**1.5 A rod is moving lengthwise with constant velocity  $v$  relative to the inertial reference frame K. At what velocity value ( $v$ ) will the length of the rod in this frame equal half of its proper length?**

- A)  $\frac{1}{2}c$  where  $c$  is the speed of light in a vacuum
- B)  $\frac{3}{4}c$

- C)  $\frac{\sqrt{3}}{2}c$   
D)  $\frac{2}{3}c$

**Thermodynamics and molecular physics**

**1.6 How will the root mean square velocity of ideal gas particles change if the gas temperature has increased fourfold?**

- A) It will increase fourfold  
B) It will decrease fourfold  
C) It will not change  
**D) It will increase twofold**

**1.7 How does the density of an ideal gas change if its temperature and pressure are doubled simultaneously?**

- A) increases fourfold  
B) decreases fourfold  
**C) does not change**  
D) halves

**1.8 Calculate the change in internal energy of the gas in a system that performs 25 kJ of work and absorbs 55 kJ of heat.**

- A) -30 kJ  
**B) +30 kJ**  
C) +80 kJ  
D) -80 kJ

**1.9 Calculate the entropy increment of a system in a reversible isothermal process at 300 K, given that 30 kJ of heat has been supplied to the system.**

- A) -100 J/K  
B) 0.01 J/K  
**C) 100 J/K**  
D) - 10 J/K

**Electromagnetism**

**1.10 A sphere is uniformly charged throughout its volume. The center of the sphere is located at a distance from a uniformly charged infinitely thin plane sheet. This distance is  $d$ . How will the force of interaction between the sphere and the plane change if this distance is doubled?**

- A) The force will double.  
B) The force will halve.  
C) The force will decrease fourfold.  
**D) The force will not change.**

**1.11 How will the inductance of a coil change if its length is doubled?**

- A) It will double.  
**B) It will halve.**  
C) It will quadruple.  
D) It will decrease fourfold.

**1.12** The area of the plates of a parallel-plate charged capacitor was reduced threefold, while the distance between the plates was increased by a factor of three, with the voltage between the plates remaining constant. How did the *modulus* of the electrostatic interaction forces between them change?

- A) It decreased by a factor of 3.
- B) It decreased by a factor of 27.**
- C) It decreased by a factor of 81.
- D) It increased by a factor of 3.

**1.13** The distance between two small circuits carrying an electrical current is much greater than their size. By what factor will the force of the interaction between the circuits decrease if this distance is doubled?

- A) 2
- B) 4
- C) 8
- D) 16**

### Optics and waves

**1.14** A beam of natural light is incident at an angle of  $55^\circ$  on a flat boundary between two media. What is the angle of refraction if the reflected beam is linearly polarized?

- A)  $15^\circ$
- B)  $25^\circ$
- C)  $35^\circ$**
- D)  $45^\circ$

**1.15** An object is placed at a distance of 7 cm from a thin converging lens, on its optical axis. Calculate the linear magnification if the focal length of the lens is 5 cm.

- A) 1.5
- B) 2
- C) 2.5**
- d. 3

**1.16** A plane electromagnetic wave with an amplitude of  $E_0$  is incident normally on a plane-parallel transparent plate. Calculate the reflectance (power reflection coefficient)  $R$  if the amplitude of the transmitted wave is  $E_0/2$ .

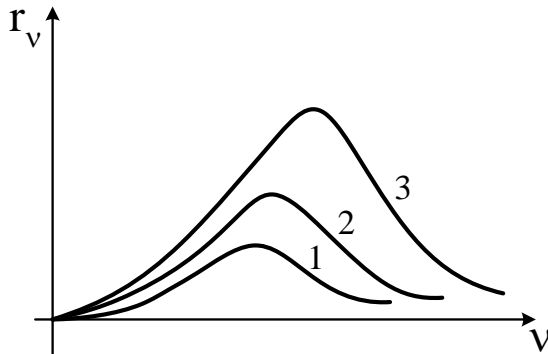
- A) 1/4
- B) 1/2
- C) 3/4**
- D) 1

**1.17** Which expression describes a monochromatic wave?

- A)  $E_0 \cos^2(\omega t)$
- B)  $E_0 \cos\left[\omega t + (t/\tau)^2\right]$
- C)  $E_0 \cos(\omega t) \sin(\omega t)$
- D)  $E_0 \exp(-t/\tau) \cos(\omega t)$

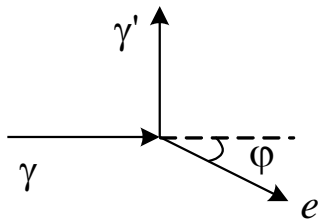
### Atomic and nuclear physics

1.18 The figure shows the dependence on frequency of the spectral exitance  $r_{\nu}$  of a blackbody. Which of the curves corresponds to the highest temperature?



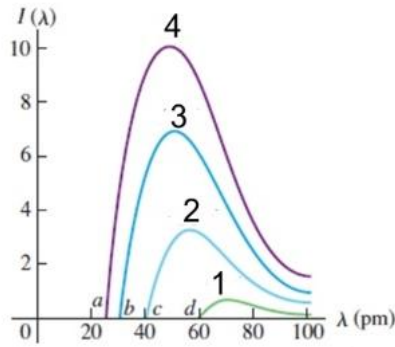
- A) 1
- B) 2
- C) 3
- D) All the curves correspond to the same temperature

1.19 In the Compton effect, photon scattering on an electron is observed. The figure shows the directions of the incident photon ( $\gamma$ ), the scattered photon ( $\gamma'$ ), and the recoiling electron ( $e$ ). The angle between the directions of the scattered photon and the incident photon is  $90^\circ$ , and that between the direction of the recoiling electron motion and the incident photon is  $\varphi = 30^\circ$ . What is the scattered photon momentum if the momentum of the incident photon is  $P_{\gamma}$ ?



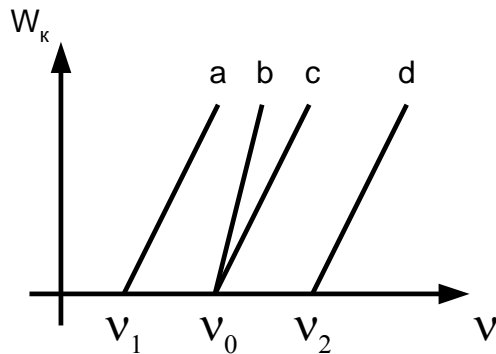
- A)  $P_{\gamma} / \sqrt{3}$
- B)  $0.5 \cdot P_{\gamma}$
- C)  $\sqrt{3} \cdot P_{\gamma}$
- D)  $1.5\sqrt{3} \cdot P_{\gamma}$

1.20 The figure shows the wavelength dependence of radiant intensity from an X-ray tube at different accelerating voltage values. Select the curve corresponding to the highest value of X-ray tube voltage.



- A) 1
- B) 2
- C) 3
- D) 4

**1.21** The dependence of maximum kinetic energy of photoelectrons on light frequency was studied by performing external photoelectric effect experiments. Line c represents the dependence of photoelectron energy  $W_K(\nu)$  for a given cathode material. Which line corresponds to the dependence of the photoelectron energy  $W_K(\nu)$  when the cathode material is replaced by a material with a higher work function?



- A) line d
- B) line a
- C) line b
- D) line c will not change in this case

**Module 2. Tasks with a numerical answer (each worth 4 points)**

**Mechanics**

**2.1** A particle moves along the x-axis. The projection of its velocity on this axis depends on time  $t$  as  $v_x = a - bt$ , where  $a = 1 \text{ m/s}$  and  $b = 2 \text{ m/s}^2$ . Calculate the distance traveled by the particle in the first second of movement. Give your answer in meters, rounded to the first decimal place.

**Answer: 0.5**

**2.2** Find the pressure force exerted by water in a reservoir with a depth of  $h = 50$  meters on a dam measuring  $L = 400$  meters, with water density  $\rho = 1000 \text{ kg/m}^3$  and gravitational acceleration  $g = 10 \text{ N/kg}$ . Give your answer in GN.

**Answer: 5**

### Thermodynamics and molecular physics

**2.3** Suppose  $\eta_0$  is the ratio between the molecular concentration of hydrogen and nitrogen at the Earth's surface, while  $\eta$  is the corresponding ratio at the height  $h = 1500$  m. Find the ratio  $\eta/\eta_0$  at room temperature,  $T = 300$  K, assuming that temperature and gravitational acceleration  $g = 9.8 \text{ m/s}^2$  do not change with height. The molar masses of hydrogen and nitrogen are  $M_1 = 2$  g/mole and  $M_2 = 28$  g/mole, respectively. The universal gas constant is  $R = 8.314 \text{ J/(K} \cdot \text{mole)}$ . Round your answer to the second decimal place.

**Answer: 1.17**

**2.4** A vessel, from which air is rapidly pumped out, contains a small amount of water at a temperature of  $0^\circ\text{C}$ . Due to intense evaporation, gradual freezing of the water occurs. What part of the initial amount of the water can be converted into ice in this way? The specific latent heat of fusion of ice and the vaporization of water are  $336$  and  $2500 \text{ J/g}$ , respectively. Give your answer in percentages, rounded to the nearest integer.

**Answer: 88**

### Electromagnetism

**2.5** A point electric dipole is located at some distance from a point charge and is oriented along a straight line connecting the dipole and the charge. In absolute values, by how many times will the force acting on the dipole decrease if the dipole is rotated by  $90$  degrees?

**Answer: 2**

**2.6** A direct current flows through a long circular coil wound uniformly. Inside the coil, there is a coaxial core of a circular cross-section of the same length. The radius of the coil is three times the radius of the core. The magnetic permeability of the core is  $1000$  times the vacuum permeability. By how many times is the magnetic energy concentrated in the core greater than the magnetic energy outside it? Fringe effects should be neglected.

**Answer: 125**

### Optics and waves

**2.7** The distance between diffraction grating grooves is  $d = 4 \mu\text{m}$ . Light with the wavelength  $\lambda = 0.55 \mu\text{m}$  is incident normally on the grating. What is the highest order of diffraction maximum possibly observed?

**Answer: 7**

**2.8** A narrow beam of X-ray radiation of wavelength  $62 \text{ pm}$  penetrates a  $2.6 \text{ cm}$  thick aluminum screen. What thickness of a lead screen is required to achieve the same level of beam attenuation? The mass absorption coefficients of aluminum and lead for this radiation are  $3.48$  and  $72.0 \text{ cm}^2/\text{g}$ , respectively. Give your answer in millimeters, rounded to the first decimal place.

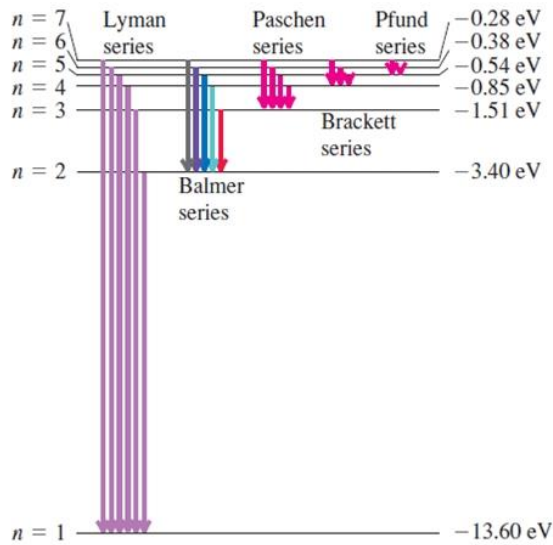
**Answer: 0.3**

### Atomic and nuclear physics

**2.9** The electron in a hydrogen atom is in a state with the principal quantum number  $n=2$ . Find the number of different electron states corresponding to the given  $n$ , taking into account the spin of the electron.

**Answer: 8**

2.10 The figure shows a diagram of energy levels in the hydrogen atom and transitions corresponding to various series.



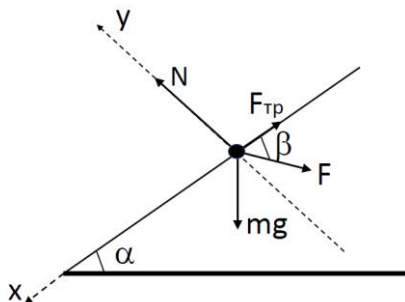
Calculate the longest wavelength of the line in the Lyman series. Give the answer in nm, rounded to the nearest integer. Planck's constant is  $h=4.136 \cdot 10^{-15}$  eV·s; the speed of light is  $c = 3 \cdot 10^8$  m/s.  
**Answer: 122**

**Module 3. Constructed response tasks (the maximum score for a correct and reasoned solution to each problem is 13 points; points will also be awarded for a partial solution, proportional to its coverage of the complete solution).**

**Mechanics**

3.1. A sledge of the mass  $m = 10$  kg is placed on an ice slide with the slope angle  $\alpha = 30^\circ$ . The coefficient of friction between the sledge and the surface is  $\mu = 0.1$ . What is the minimum force required to keep the sledge in equilibrium? The free fall acceleration is  $g = 9.8$  m/s<sup>2</sup>.  
 Give the answer in kN, rounded to the second decimal place.

**Solution:**



Since  $\mu < \tan \alpha$ , the sledge cannot be in mechanical equilibrium. To keep the sledge in equilibrium, force  $\mathbf{F}$  must be applied at an angle  $\beta$  to the tilted plane to prevent the sledge from sliding. The balance of forces providing equilibrium is

$$\mathbf{F} + m\mathbf{g} + \mathbf{N} + \mathbf{F}_{mp} = 0, \tag{3.1.1}$$

where  $\mathbf{F}_{mp}$  is the maximum static friction force, and  $\mathbf{N}$  is the normal reaction force.

**(4 points)**

For the x- and y-axis projections of equation 3.1.1, which assumes that  $F_{mp} = \mu N$ , we obtain

$$mg \sin \alpha - F \cos \beta - \mu N = 0, \quad (3.1.2)$$

and

$$N - mg \cos \alpha - F \sin \beta = 0, \quad (3.1.3)$$

respectively.

From equations 3.1.2 and 3.1.3, we have

$$F = \frac{\sin \alpha - \mu \cos \alpha}{\cos \beta + \mu \sin \beta} mg. \quad (3.1.4)$$

**(4 points)**

It follows from expression 3.1.4 that force  $F$  is at its minimum when the value of the denominator  $\varphi(\beta) = \cos \beta + \mu \sin \beta$  is maximum. The function  $\varphi(\beta)$  has the maximum value if

$$\operatorname{tg} \beta = \mu. \quad (3.1.5)$$

**(2 points)**

Consequently, the minimal value of the force is

$$F = \frac{\sin \alpha - \mu \cos \alpha}{\sqrt{1 + \mu^2}} mg \quad (3.1.6)$$

**(1 point)**

Hence,  $F \approx 0.04 \text{ kN}$

**(2 points)**

### Thermodynamics and molecular physics

**3.2 An ideal gas is expanded isothermally from a volume of  $0.1 \text{ m}^3$  to  $0.3 \text{ m}^3$ . The final gas pressure is  $2 \cdot 10^5 \text{ Pa}$ . Determine the amount of heat transferred to the gas during this process. Give your answer in kJ, rounded to an integer.**

**Solution:** According to the first law of thermodynamics, the amount of heat supplied to the gas is

$$Q = \Delta U + W. \quad (3.2.1)$$

Since the process is isothermal, the increment of internal energy  $\Delta U = 0$ ; therefore, the problem can be reduced to calculating the work done by the gas when expanding from volume  $V_1 = 0.1 \text{ m}^3$  to volume  $V_2 = 0.3 \text{ m}^3$

$$Q = W. \quad (3.2.2)$$

**(2 points)**

This work is

$$W = \int_{V_1}^{V_2} p dV, \quad (3.2.3)$$

where,  $p$  is the pressure of the gas as a function of its volume.

**(2 points)**

This function  $p(V)$  is given by the equation of an ideal gas state

$$p = \frac{\nu RT}{V}. \quad (3.2.4)$$

**(2 points)**



Substituting (3.2.4) into (3.2.3) and taking into account that the amount of substance  $\nu$  and the temperature  $T$  in the gas remain constant during the process, we obtain

$$W = \nu RT \int_{V_1}^{V_2} \frac{dV}{V} = \nu RT \ln \left( \frac{V_2}{V_1} \right) \quad (3.2.5)$$

**(3 points)**

Isolating the unknown  $\nu RT$  from the gas state equation through the final pressure  $p_2 = 2 \cdot 10^5$  Pa and the final volume  $V_2 = 0.3$  m<sup>3</sup>, we obtain

$$W = p_2 V_2 \ln \left( \frac{V_2}{V_1} \right) \quad (3.2.6)$$

**(2 points)**

Hence,

$$W \approx 66 \text{ kJ} \quad (3.2.7)$$

**(2 points)**

### Electromagnetism

3.3 Two identical empty parallel-plate capacitors are half-filled with the same dielectric. The first one is filled in such a way that the vacuum-dielectric interface is perpendicular to the capacitor plates; the second, so that the vacuum-dielectric interface is parallel to the plates. The capacitance of the first capacitor is 25/16 times that of the second capacitor. What is the permittivity  $\varepsilon$  of the dielectric? Fringe effects should be neglected.

**Solution:** Let  $C$  be the capacitance of an empty capacitor. The first capacitor can be considered as two capacitors connected in parallel, whose capacitance is  $C/2$  and  $\varepsilon C/2$ , respectively **(3 points)**. Thus, the capacitance of the first capacitor is

$$C_1 = \frac{(\varepsilon + 1)}{2} \cdot C \quad (3.3.1)$$

**(2 points).**

The second capacitor can be represented as two capacitors connected in series, their capacitance equal to  $2C$  and  $2\varepsilon C$ , respectively **(3 points)**. Then, the capacitance of the second capacitor can be calculated as

$$\frac{1}{C_2} = \frac{1}{2C} + \frac{1}{2\varepsilon C} = \frac{(\varepsilon + 1)}{2\varepsilon} \cdot \frac{1}{C} \quad (3.3.2)$$

**(2 points).**

By multiplying these two equations, we obtain

$$\frac{25}{16} = \frac{(\varepsilon + 1)^2}{4\varepsilon} \quad (3.3.3)$$

Hence,  $\varepsilon = 4$  **(3 points)**.